# Activism, Stock Selection, and Indexing in Equilibrium *

Steven D. Baker† David A. Chapman‡
Michael F. Gallmeyer§

April 3, 2023

**Abstract**

We construct a dynamic general equilibrium production economy intermediated by two types of actively managed funds and a passive index. "Activists" and "stock selectors" are both socially valuable, albeit in different ways. Households allocate their wealth across the different funds, determining the endogenous composition of the intermediation sector. We characterize the equilibrium impact of changes in search costs, productive efficiency, and index fees on the composition of the fund industry. Our general equilibrium perspective adds new insights in the debate about active versus passive management. While a decrease in the passive index fee benefits households through cheaper diversification, it also impacts the composition of the managed fund industry which can raise or lower investor welfare.

*We would like to thank Sebastien Betermier (discussant), Matthijs Breugem (discussant), Rich Evans, Simon Gervais (discussant), Christian Heyerdahl-Larsen, Wei Li (discussant), Andrey Malenko (discussant), Jonathan Reuter, Savitar Sundaresan (discussant), Fernando Zapatero (discussant), Yao Zeng (discussant), conference participants at the European Finance Association Conference, the Finance Down Under Conference, the FIRS Finance Conference, the Four Corners Conference, the ITAM Finance Conference, the Johns Hopkins Carey School Finance Conference, the Northern Finance Association Conference, and the Western Finance Association Conference, and seminar participants at the University of Iowa, the University of Melbourne, University of Luxembourg, the Federal Reserve Bank of Dallas, and Federal Reserve Bank of Richmond for comments. An earlier version of the paper was titled "Activism and Indexing in Equilibrium." The views expressed in this article are solely those of the authors. They do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

†The Federal Reserve Bank of Richmond, `Steven.Baker@rich.frb.org`.

‡The McIntire School of Commerce, University of Virginia, `david.chapman@virginia.edu`.

§The McIntire School of Commerce, University of Virginia and the Faculty of Business and Economics, University of Melbourne, `mgallmeyer@virginia.edu`.

# 1 Introduction

If a firm's managers are destroying shareholder value, an asset manager faces two primary choices: voice or exit. Voice, or activism, is generally understood to describe a broad range of shareholder actions designed to influence a firm's real or financial policy decisions. As such, it is recognized as one important component of corporate governance. We build a general equilibrium model of the asset management industry where the composition of the industry dynamically changes. Our goal is to understand how activism coexists with alternative asset management channels such as active stock selectors and passive index funds. We want to understand the implications of fee compression for household welfare and for the relative sizes of the different fund channels, where fee compression is defined in the financial press as the reduction in active management fees coincident with increased passive management.

The foundation of the model is a set of productive technologies representing industries. A representative household can invest in these technologies through three different types of intermediaries: a passive index, activists, and stock selectors. The passive fund allows the representative household access to an equal-weighted portfolio of all industries at low or zero fees. Its existence captures the intuition that the household does not require a skilled manager to spread capital evenly across the equity market, but the household does require a skilled manager to identify the most efficient investments.

An activist creates a fund by incurring a search cost to match with an industry, exercising voice, and forcing the industry to use its capital efficiently. The industry remains efficient as long as it is matched with an activist. This is the direct benefit of voice. However, the activist also generates a positive externality because the increase in industry efficiency contributes to increased performance for the passive index. A stock selector, at times referred to as just a selector, invests in an industry via an active strategy that resembles security selection within an industry. This security selection allows the stock selector fund to generate high cash flows from an industry without generating a spillover to the other investors in that industry. Both activists and stock selectors choose fees optimally in a symmetric Nash equilibrium, given the state of competition in the managed fund market.

The household determines the demand for each of the three types of funds by solving an optimal consumption and portfolio choice problem. In equilibrium, the markets for goods and fund shares clear, determining the aggregate size of each fund sector. We examine the key factors determining the interactions among agents through a set of numerical experiments. We focus on how fee structures and changes in the active investors' technologies dynamically impact the relative sizes of the fund management sectors, and how the composition of the active management sector affects household welfare.

For a wide range of parameter values, a non-degenerate stationary distribution for each fund type exists in the model's steady state. Our baseline parameterization demonstrates how fund market competition impacts assets under management and household welfare through both diversification and spillover effects. Although the equilibrium characteristics of activists and stock selectors are similar in many ways, their responses to changes in the state variables are not necessarily identical. For example, due to the nature of their investing process, stock selectors can displace incumbent activists by investing in the same industries. We later see in our dynamic analysis that this introduces a "predator/prey" component to the dynamics of the relative sizes of the activist and stock selector sectors that can lead to interesting interaction effects.

But the absence of spillovers from stock selectors can also create space for activists, by making activism more profitable. For example, as the number of activist funds grows, the household invests more in the activist sector overall as this sector becomes better diversified. However, more activist funds also make the passive index more efficient, leading to a spillover effect that eventually makes assets under management in the active sector fall. Stock selectors do not generate a spillover effect. If more of them are in the market, activist assets under management can actually rise as activist funds are better complements to stock selector funds than the passive fund in the household's portfolio.

This dynamic between activists and stock selectors also has a differential impact on the present value of fees collected. As activists funds increase in number, the present value of both activist fees and stock selector fees fall due to increased competition from the index. However, as the number of stock selectors increase, the present value of both activist fees and stock selector fees rise. This non-intuitive result is driven by stock selector funds not generating a spillover so the passive index is not a cheaper investment product to achieve the same cash flows.

Declining passive index fees and fee compression have become commonplace in the asset management industry.[1] Understanding their impact on investor welfare requires a dynamic assessment in general equilibrium of their effect on the incentives for activists to participate in the asset management industry. Our model can endogenously generate a pattern of falling fund fees and active management fund dynamics, as recently seen in the fund management industry, without any changes in the model's structural parameters. A decrease in the passive index fee, a structural parameter in our model, provides the benefit of cheaper diversification options, but it can be harmful to overall efficiency if the reduction in fees reduces the number of activists willing to participate in the fund management industry. We demonstrate this process in a simple example that curtails stock selector entry. Without restricting stock selector entry, a decrease in passive index fees is welfare improving to households for both lower fees and a reconfiguration of the composition of the managed money sector, as activists fare better in low fee environments than do stock selectors.

Finally, we explore how changes in fund management technology impact fund competitiveness through search and monitoring and/or stock selection success. When activists face

[1] See for example Morningstar's Annual Fund Fee Study available at `https://www.morningstar.com/lp/annual-us-fund-fee-study`.

lower search costs for new investments, households are better off. Lower search costs for stock selectors, however, lead to higher fees and more money managed in selector funds to the detriment of households. Similar results hold when varying monitoring costs for activists or stock selection success for selectors.

The conventional focus of mutual fund and hedge fund research has been on whether or not active management delivers “alpha” through the selection of mis-priced securities. Our purpose in emphasizing the role of voice is not to argue that the conventional focus is unimportant, but rather to argue that activism — and the interaction of activism and selection — may play a more important role in understanding asset management in general equilibrium than has generally been acknowledged. This view is consistent with the recent survey evidence in McCahery, Sautner, and Starks (2016) and the empirical literature on activism summarized in Brav, Jiang, and Kim (2015a).

The rest of the paper is organized as follows: After placing our work in the context of the relevant literature on both shareholder monitoring and active management, we present the model and characterize the equilibrium. We then examine how the equilibrium responds to changes in the cost of intermediation along several dimensions. The final section of the paper concludes. All proofs are in the appendix.

## 2 Our Contribution Relative to the Existing Literature

In order for any shareholder (or group of shareholders) to effectively monitor firm managers, they must solve a free-rider problem; i.e., the costly efforts of shareholders who monitor the firm also benefit other shareholders who exert no effort. This problem might be overcome by large shareholders via takeovers, as in Shleifer and Vishny (1986). Yet ownership that is too highly concentrated could lead to sub-optimally tight control by shareholders (Burkart, Gromb, and Panunzi, 1997), or to weaker monitoring incentives stemming from reduced liquidity and price informativeness (Holmström and Tirole, 1993). Assuming that the right number of shareholders are paying the right amount of attention, providing improved incentives to the firm's managers remains a nontrivial problem (Core, Guay, and Larcker, 2003).

The results in Admati, Pfleiderer, and Zechner (1994) provide theoretical arguments against the use of voice by demonstrating that the equilibrium level of monitoring is well below its socially optimal level, and DeMarzo and Urosevic (2006) extend this result to show that, over time, an activist will actually hold a perfectly diversified portfolio. Marinovic and Varas (2019) show that whether or not the activist monitors and whether or not that monitoring increases firm value depends critically on the presence of information asymmetry about the activist's ability.[2]

Despite these substantial incentive and information obstacles, McCahery, Sautner, and Starks (2016) present survey evidence that over half of institutional investors engage directly with the management and boards of the firms in which they hold stock, presumably to the benefit of shareholders at large. The survey article of Brav, Jiang, and Kim (2015a) provides additional empirical evidence of the impact of activism on firms. Brav, Jiang, and Kim (2015b) show, empirically, that activism has a positive impact on plant-level productivity.

We sidestep the discordance of the theoretical results with the empirical findings by abstracting from the specific details of how activists change firm policies – and by extension firm value. Instead, we assume a matching technology that pairs fund managers (both activists and stock selectors) with firms. Once the match is formed, the monitoring requires no costly effort by managers, and the representative household has perfect information about which firms are matched to each type of manager. We use voice and monitoring as a metaphor

[2]This additional layer of delegation also introduces new agency problems that have been the focus of a large literature on optimal managerial contracting; see, for example, the theoretical work of Bhattacharya and Pfleiderer (1985), Stoughton (1993), Heinkel and Stoughton (1994), Starks (1987), and the empirical work in Almazan et al. (2004).

for how active managers can deliver value to households in equilibrium in the absence of asymmetric information. We are interested in understanding the factors that affect the relative sizes of the active and passive sectors of the asset management industry and how the structure of the asset management industry affects household welfare in general equilibrium.

Pastor and Stambaugh (2012) provide an alternative explanation for the relative sizes of the active and passive sectors of the fund industry. Their focus is on explaining the poor performance of active funds relative to a passive benchmark (the "active management puzzle") in an equilibrium model where funds face decreasing returns to scale; i.e., when fund performance erodes because too many managers trade in related strategies. Their explanation of active vs. passive sector sizes is complementary to ours. Their model is more general than ours in that investors can learn about managerial skill while our framework assumes complete information. Our model is more general than theirs in that we incorporate a production economy and our household explicitly solves a multiperiod (infinite-horizon) portfolio problem.

Whereas we focus on voice as a source of positive spillovers from active management, Buss and Sundaresan (2020) focus on improved price informativeness as a source of positive spillovers from active management. In theoretical and empirical examples, they document a surprising positive relationship between high passive ownership and price informativeness. Their argument is that high passive ownership increases the incentive for information production by active managers. In our setting, passive and active fund shares are dynamic and endogenously determined, so the relationship is nuanced. When passive ownership is high because index funds charge low fees, activists and selectors compete by reducing fees too, which can benefit households overall despite reduced monitoring of firms by activists.

Complementary to our work, Gervais and Strobl (2021) also study a model of fund management where active fund managers impact production processes through the allocation of funds across firms. Their work, in a noisy rational expectations setting, studies a different question from ours. They explore how performance benchmarking for active managers is confounded by their real impact on the economy. They then demonstrate that it is still optimal to invest with these active managers even when their performance lags the market portfolio. Both their work and ours show that welfare can be improved by the presence of active fund managers who impact a firm's production process.

In contrast to our work that studies the equilibrium dynamics of the size of the actively managed and passively managed sectors, Gârleanu and Pedersen (2018, 2019) and Corum, Malenko, and Malenko (2022) study the impact on market efficiency of endogenous costly information acquisition in a noisy rational expectations equilibrium with one period of trade. Gârleanu and Pedersen (2018) explore how market efficiency is impacted when investors incur search costs to find asset managers. Gârleanu and Pedersen (2019) extend this work by focusing on the impact on passive investing when there is a fixed number of active asset managers. Corum, Malenko, and Malenko (2022) build on both of these works by adding a governance choice to both active and passive managers in a setting where investors are risk neutral. They show that increased passive investing can have an ambiguous impact on the level of governance. Given we assume long-lived risk averse households as well as entry and exit of fund managers, we are able to explore fund management sector dynamics and household welfare that are driven by spillover and diversification effects.

We assume that passive managers provide diversification services at low cost, and they do not engage in any use of voice or (by definition) other active strategies. Azar, Schmalz, and Tecu (2018) argue that even passive managers can facilitate collusion in concentrated industries, to the detriment of households. However, there is a growing literature that contradicts the findings in Azar, Schmalz, and Tecu (2018); see, for example Dennis, Gerardi, and Schenone (2022).

# 3 Model

We study a continuous-time, infinite-horizon economy in which the fund management industry intermediates between households and productive investment opportunities. We build the model from the bottom up, beginning with the productive technologies, or industries. Investment managers who are either activists or stock selectors search for opportunities to match with industries, thereby forming funds. A lower cost passive index fund also exists which invests equally in all industries. Finally, a representative household optimally allocates capital across funds. Having closed the model, we solve and analyze fund and capital market dynamics in general equilibrium.

## 3.1 Industries

Capital may be productively invested in $N$ industries. All industries have an identical productivity of capital $\mu$, such that an industry $i$ with capital $K_{i,t}$ produces output at gross rate $\mu K_{i,t}dt$. The capital depreciation process for industry $i$ is

$$K_{i,t}\left[-\delta_{i,t}dt + \overline{\sigma}d\overline{W}_t + \sigma dW_{i,t}\right]. \qquad (1)$$

The Brownian motion $\overline{W}_t$ captures a capital depreciation shock common to all industries, whereas the Brownian motion $W_{i,t}$ captures industry-specific depreciation, which is independent of $\overline{W}_t$ or $W_{i',t}$, $i' \neq i$.

At any given time, the deterministic depreciation rate $\delta_{i,t}$ takes one of two values. When an industry is in an activist fund, depreciation takes a smaller value $\delta_{i,t} = \delta_A$, reflecting real efficiency gains from monitoring. An industry that is not part of an activist fund is assumed to be inefficiently run, with $\delta_{i,t} = \delta > \delta_A$. This simple reduced-form specification captures the idea that activism produces a positive externality, or spillover, through monitoring. We model an active manager's "voice" in reduced form, but we have in mind mechanisms for voice as discussed in the survey evidence in McCahery, Sautner, and Starks (2016).[3]

For the most part, we can treat industries as the most basic level of productive technology in our model. However, in Appendix A we further subdivide each industry into a continuum of firms. This subdivision rationalizes the following assumptions about the interaction of stock selectors and industries.

First, if a selector fund invests in industry $i$, it achieves an effective depreciation rate $\delta_S < \delta$, but without altering the depreciation rate achieved by other investors in that industry. Appendix A rationalizes this outcome as a result of reduced-form security selection that produces no spillovers for the industry.

Second, if a stock selector invests in industry $i$, then it displaces any activist currently invested in industry $i$. The only fund type that may simultaneously invest in that industry is the index, which achieves depreciation rate $\delta$ on industry $i$ investment. Appendix A rationalizes activist exit following selector entry as the result of arbitrage opportunities that arise between selector and activist funds within the same industry, which have the effect of driving the activist's assets under management (AUM) to zero in that industry.

## 3.2 Funds

Ex-ante identically skilled managers search for opportunities in the labor market for asset managers. Managers match with industries to form investment funds, which may be either selector funds or activist funds. A fund consists of one industry and one manager. An unemployed (potential) manager may search for opportunities as either an activist or a stock selector, or may choose not to search at all. The choice is sensitive to the current

[3]We also assume that activism is always successful. As noted earlier, reality dictates that activism will not always be successful and can be curtailed by free rider problems (Grossman and Hart, 1980; Shleifer and Vishny, 1986) or liquidity issues (Coffee, 1991). Recent empirical work on activism includes Becht et al. (2017) and Boyson and Pichler (2018).

competitiveness of the fund market. At any moment, there are $a_t$ incumbent activist funds and $s_t$ incumbent selector funds. Potential managers who choose not to search can be thought of as remaining in the general household pool, earning reservation utility with certainty equivalent value $\underline{c}K_t$, where $K_t$ is the aggregate capital stock, or equivalently aggregate wealth.[4]

If a potential manager chooses to search, that manager pays a flow cost $\zeta_A K_t dt$ while searching for activist opportunities or $\zeta_S K_t dt$ for stock selector opportunities. New matches are formed at rates

$$\hat{a}_t^{1-\nu}(N - a_t - s_t)^{\nu}, \quad \hat{s}_t^{1-\nu}\left(\eta(N - a_t - s_t) + (1-\eta)a_t\right)^{\nu}, \tag{2}$$

for activists and selectors, respectively, where $\hat{a}_t$ is the number of potential managers searching for activist opportunities and $\hat{s}_t$ the number of potential selectors searching. In our experiments, we assume that the elasticity parameter $\nu = 1/2$ and that there is no difference in $\nu$ between activists and stock selectors. This corresponds to a prior belief that matching is equally difficult for both manager types. However, there is a difference between activists and selectors in the composition of candidate industries for matching, as potential activists match only with industries without incumbent funds, whereas potential selectors may also match with industries having activist incumbents. This gives rise to a predator-prey dynamic between stock selectors and activists, with parameter $\eta \in [0, 1]$ modulating the strength of that dynamic. When $\eta = 1$, selectors cannot displace incumbent activists, whereas when $\eta = 0$ stock selectors are only able to match with industries having incumbent activists. In our baseline parameters we choose $\eta = 1/2$. The constant returns to scale matching technology that we use is standard in the broader labor literature; see for example Petrongolo and Pissarides (2001), Rogerson, Shimer, and Wright (2005), and Petrosky-Nadeau, Zhang,

[4]The number of potential managers is not generally important to the setup, but we assume that there are a large number of potential managers relative to industries.

and Kuehn (2018).

Constant returns to scale implies that individual potential activist and stock selector managers find matches at the respective rates

$$\left(\frac{N - a_t - s_t}{\hat{a}_t}\right)^{\nu}, \quad \left(\frac{\eta(N - a_t - s_t) + (1-\eta)a_t}{\hat{s}_t}\right)^{\nu}. \tag{3}$$

The chance a searching manager succeeds depends on the tightness of the labor market, a ratio of available target industries to searching managers. For most of our analysis, the rate at which new selector funds match with incumbent activists is also the rate at which incumbent activist funds dissolve.[5] Finally, selector funds dissolve at an exogenous rate $\theta_S$ individually, or $\theta_S s_t$ in the aggregate.

Potential managers take the present value of their earnings as given when making search decisions. An incumbent activist manager earns fee income with time $t$ present value $\Phi_A(a_t, s_t, K_t)$. An incumbent stock selector manager has present value of expected earnings $\Phi_S(a_t, s_t, K_t)$. Fee income depends on funds under management, which is determined endogenously as a part of the household's equilibrium asset allocation.

Potential managers will enter the labor market (search) until the net present value of a new fund is zero. For activists, $\hat{a}_t$ is the number of searchers[6] such that

$$(\Phi_A(a_t + 1, s_t, K_t) - \underline{c}K_t)\left(\frac{N - a_t - s_t}{\hat{a}_t}\right)^{\nu} = \zeta_A K_t. \tag{4}$$

When a potential activist matches with a target industry, that activist exchanges fee revenues for his household consumption stream. The exchange must be favorable enough, and the

[5]We allow for the possibility that some activists also exit for exogenous reasons, at rate $\theta_A$ individually or $\theta_A a_t$ in the aggregate. We set $\theta_A = 0$ in our baseline parameters.

[6]We allow $\hat{a}_t$ to take nonnegative real values, rather than restricting it to nonnegative integer values, hence Equation (4) holds with equality. $\hat{a}_t$, and $\hat{s}_t$ in Equation (5), can be interpreted more generally as search intensities. As a practical matter, continuously-valued search intensities facilitate convergence of the numerical solution routine.

intensity of matching high enough, to justify the search costs.

The equivalent condition for stock selectors is

$$\left(\mathbb{E}\left[\Phi_S(a', s_t+1, K_t)|(a_t, s_t)\right] - \underline{c}K_t\right)\left(\frac{\eta(N-a_t-s_t)+(1-\eta)a_t}{\hat{s}_t}\right)^{\nu} = \zeta_S K_t, \tag{5}$$

where the conditional expectation over fees incorporates the probabilities of the new stock selector matching either an activist or an exclusively index industry.

We verify that one solution to the activist and stock selector fee-setting problems has the property that the present value of fees is homogeneous of degree one in capital $K_t$, such that they can be written $\Phi_A(a_t, s_t, K_t) = \phi_A(a_t, s_t)K_t$ for activists, and $\Phi_S(a_t, s_t, K_t) = \phi_S(a_t, s_t)K_t$ for selectors. This allows us to solve for $\hat{a}_t$, the number of potential managers seeking to start activist funds, such that

$$\hat{a}_t = \left(\frac{\phi_A(a_t+1, s_t) - \underline{c}}{\zeta_A}\right)^{\frac{1}{\nu}}(N - a_t - s_t), \tag{6}$$

whereas $\hat{s}_t$, the number of potential managers searching to be selectors, satisfies

$$\hat{s}_t = \left(\frac{\mathbb{E}\left[\phi_S(a', s_t+1)|(a_t, s_t)\right] - \underline{c}}{\zeta_S}\right)^{\frac{1}{\nu}}(\eta(N-a_t-s_t)+(1-\eta)a_t). \tag{7}$$

Finally, a third type of investment fund exists alongside stock selectors and activists: a passive index fund, which invests equally in each of the $N$ industries. This fund is unique, requires no manager, and charges a small, exogenous, and constant fee rate $\bar{\phi}_I$ proportional to capital under management.[7] In addition to capturing competition from low-cost index funds, the passive fund reflects the idea that households do not require skilled managers in order to spread capital evenly across all industries, but they do require skilled managers to identify the most efficient investments. In a simple way, this captures the idea that fund

[7] In our baseline parameterization, $\bar{\phi}_I = 0$.

managers are better informed than households.

### 3.3 Households

An infinitely lived representative household allocates capital to investment funds in order to maximize expected lifetime utility. While the state of competition among funds matters to households, there is no reason for households to differentiate between funds of a given type. For example, each activist charges an identical fee proportional to capital, and each invests in a unique — but equivalent — efficient industry. Hence it is optimal, for purposes of diversification, for the household to invest the same amount in each activist fund. The relevant question is how much the household should invest in activist funds *collectively*.

Without loss of generality, let the first $a_t$ industries be activist funds, the next $s_t$ industries be stock selector funds, and the remaining $N - a_t - s_t$ industries belong exclusively to the the passive index.[8] We define composite Brownian motions

$$W_{A,t} = \sum_{j=1}^{a_t} W_{j,t}, \quad W_{Q,t} = \sum_{j=a_t+1}^{a_t+s_t} W_{j,t}, \quad W_{I,t} = \sum_{j=a_t+s_t+1}^{N} W_{j,t}, \tag{8}$$

representing risk specific to activists, selectors, and the passive fund, respectively.

While we have in mind that the passive fund fee $\bar{\phi}_I$ is small, a fraction $\frac{N-a_t}{N}$ of its holdings is inefficient, with higher depreciation rate $\delta$. Both activist and stock selector funds offer higher efficiency in the sense that their investments have lower depreciation rates: $\delta_A$ for activists and $\delta_S$ for selectors. However these funds also charge commensurately higher fees proportional to capital under management. Fee rates may vary depending on the amount of competition in the fund market. The stock selector's fee rate is $\bar{\phi}_{S,t} > 0$, and the activist's fee rate is $\bar{\phi}_{A,t} > 0$. Households and potential managers take these fees as given. Later we

[8] Recall that funds consist of non-overlapping industry-manager pairs but the passive index invests equally in every industry.

explain how incumbent managers strategically set fees to maximize revenues.

Households allocate aggregate capital, $K_t$, across the three types of funds, and also choose the consumption-capital ratio $c_t$. Let $\pi_{A,t}$ be the activists' fraction of capital, $\pi_{S,t}$ the stock selectors' fraction of capital, and the residual $1-\pi_{A,t}-\pi_{S,t}$ is invested in the passive index. The aggregate capital accumulation process is

$$\begin{aligned}\frac{dK_t}{K_t} =&(\mu - c_t)dt + \overline{\sigma} d\overline{W}_t \\ &+ \begin{bmatrix} \pi_{A,t} \\ \pi_{S,t} \\ (1-\pi_{A,t}-\pi_{S,t}) \end{bmatrix} \cdot \begin{bmatrix} -(\bar{\phi}_{A,t}+\delta_A)dt + \frac{\sigma}{\sqrt{a_t}} dW_{A,t} \\ -(\bar{\phi}_{S,t}+\delta_S)dt + \frac{\sigma}{\sqrt{s_t}} dW_{Q,t} \\ -(\bar{\phi}_{I}+\delta_{I,t})dt + \frac{\sigma}{N}\left(\sqrt{N-a_t-s_t}dW_{I,t} + \sqrt{a_t}dW_{A,t} + \sqrt{s_t}dW_{Q,t}\right) \end{bmatrix},\end{aligned} \tag{9}$$

where

$$\delta_{I,t} = \frac{1}{N}\left((N-a_t)\delta + a_t\delta_A\right) \tag{10}$$

is the mean depreciation rate of industries in the index. Since $\delta_A < \delta$, Equation (10) captures positive spillovers from activism: the more activists funds $a_t$, the more industries are efficient, which also benefits index investors through smaller index depreciation $\delta_{I,t}$. The number of stock selector funds $s_t$ has no direct effect on index depreciation as selectors produce no spillovers.

Households have CRRA utility, and choose consumption and a capital allocation to maximize their expected utility,

$$\max_{\{c_t,\pi_{A,t},\pi_{S,t}\}_{t=0}^{\infty}} E_0\left[\int_0^{\infty} e^{-\rho t}\frac{(c_tK_t)^{1-\gamma}}{1-\gamma}dt\right], \tag{11}$$

subject to Equation (9).

# 4 Equilibrium

The economic state is summarized by the tuple $\omega_t = (s_t, a_t)$, a Markov process capturing current competition in the fund market based on the number of stock selectors and activists, as well as the aggregate capital stock $K_t$. Since the economy is time-homogeneous, we omit time subscripts in the solution.

## 4.1 The Fund Managers' Problems

We begin by solving for a manager's entry decision taking fees as given. Expectation under the risk-neutral measure is denoted $\mathbb{E}^Q$, and the instantaneous risk free discount rate is $r(w)$, i.e., a function of the Markov state only. Define the space of Markov states $\Omega$, and for current state $\omega \in \Omega$, the instantaneous rate of transition to $\omega' \in \Omega$ is $\lambda_{\omega,\omega'}$, and $\Lambda$ the transition matrix, with $\lambda_{\omega,\omega} = -\sum_{\omega' \in \Omega, \omega' \neq \omega} \lambda_{\omega,\omega'}$. The equilibrium fee and a manager's entry decision are determined simultaneously, but we discuss them sequentially.

The instantaneous hazard rate $\beta_A(\omega)$, serves two distinct purposes in the model. First, it captures the possibility that a particular activist is driven out of the market. This occurs when the activist's industry becomes part of a stock selector fund in our baseline example, or because an activist exits for exogenous reasons in one of our extensions. Second, the hazard rate also limits the conditions under which potential activists will search to form new funds. It is possible for the present value of activist fees to be positive even if the current *flow* of fees is non-positive, i.e., if the household would choose a zero or short position in activist funds. To prevent this, we impose that activists immediately exit if their current fee flow turns negative. It follows that

$$\beta_A(\omega) = \begin{cases} \frac{\lambda_{(a,s),(a-1,s+1)}}{a} + \theta_A \text{ if } \pi_A(\omega) > 0, \\ \infty \text{ otherwise }. \end{cases} \tag{12}$$

The present value of an incumbent activist's fees is

$$\Phi_A(\omega_t, K_t) = \mathbb{E}_t^Q \left[ \int_t^\infty e^{\int_t^s \beta_A(\omega_v)+r(\omega_v)dv} \frac{\bar{\phi}_A(\omega_u)\pi_A(\omega_u)}{a_u} K_u du \right]. \tag{13}$$

Similarly, for stock selectors we define

$$\beta_S(\omega) = \begin{cases} \theta_S \text{ if } \pi_S(\omega) > 0, \\ \infty \text{ otherwise }, \end{cases} \tag{14}$$

and the present value of an incumbent selector's fees is

$$\Phi_S(\omega_t, K_t) = \mathbb{E}_t^Q \left[ \int_t^\infty e^{\int_t^s \beta_S(\omega_v)+r(\omega_v)dv} \frac{\bar{\phi}_S(\omega_u)\pi_S(\omega_u)}{s_u} K_u du \right]. \tag{15}$$

Using a generalized Feynman-Kac theorem, the stochastic integrals in Equation (13) and Equation (15) can be expressed as the solutions to the following partial differential equations (PDEs):

$$\mathcal{A}\Phi_A(\omega, K) - (\beta_A(\omega) + r(\omega))\Phi_A(\omega, K) + \frac{\bar{\phi}_A(\omega)\pi_A(\omega)}{a} K = 0 \tag{16}$$

and

$$\mathcal{A}\Phi_S(\omega, K) - (\beta_S(\omega) + r(\omega))\Phi_S(\omega, K) + \frac{\bar{\phi}_S(\omega)\pi_S(\omega)}{s} K = 0, \tag{17}$$

where $\mathcal{A}$ denotes the infinitesimal generator for $(\omega, K_t)$. The following proposition characterizes the managerial fees:

**Proposition 1.** *Under the assumption that $\Phi_A(\omega, K) = \phi_A(\omega)K$ and $\Phi_S(\omega, K) = \phi_S(\omega)K$, the PDEs in Equation (16) and Equation (17) can be written as*

$$\mathcal{A}\Phi_A(\omega, K) = \phi_A(\omega)(r(\omega) - c(\omega))K + \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\phi_A(\omega') - \phi_A(\omega)]K, \tag{18}$$

*and*

$$\mathcal{A}\Phi_S(\omega, K) = \phi_S(\omega)(r(\omega) - c(\omega))K + \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\phi_S(\omega') - \phi_S(\omega)]K. \tag{19}$$

*The instantaneous transition rates defining fee dynamics are*

$$\lambda_{(a,s),(a+1,s)} = \hat{a}^{1-\nu}(N - a - s)^{\nu}, \tag{20}$$

$$\lambda_{(a,s),(a-1,s+1)} = (1-\eta)a\left(\frac{\hat{s}}{\eta(N - s - a) + (1-\eta)a}\right)^{1-\nu}, \tag{21}$$

$$\lambda_{(a,s),(a,s+1)} = \eta(N - a - s)\left(\frac{\hat{s}}{\eta(N - a - s) + (1-\eta)a}\right)^{1-\nu}, \tag{22}$$

$$\lambda_{(a,s),(a-1,s)} = \theta_A a, \tag{23}$$

$$\lambda_{(a,s),(a,s-1)} = \theta_S s, \tag{24}$$

*and zero to all other states, subject to the additional restriction that transition intensity to states outside of* $\{0 \ldots N\}$ *is always zero.*

Given the entry decision, we now endogenize how fee rates are set by incumbent managers. We assume managers behave strategically, albeit in a limited sense. For each state $\omega$, we solve for a symmetric Nash equilibrium in which each manager chooses his fee rate to maximize his flow of revenues, i.e., the product of his fee rate and the household's allocation to his fund. In doing so the manager takes into account the response of the household to a potential change in fee rate, given the fees charged by the manager's competitors. However incumbent managers do not consider how their fee strategy alters fund market dynamics through its effects on the labor market search behavior of potential managers. This assumption is primarily for tractability, since it decouples the determination of fee rates from equilibrium fund market dynamics. However it seems reasonable that while fund managers might respond to current competitive pressures, they may not anticipate how their behavior will alter the evolution of the fund market in the future.

To streamline the exposition, we present solutions under the following assumption.

**Assumption 1.** *In state $\omega = (a, s)$, the representative household optimally invests a positive amount in all funds.*

For our baseline parameters, given in Table 1, states satisfying Assumption 1 account for 97% of the probability mass under the equilibrium stationary distribution. Appendix B presents solutions when Assumption 1 is violated.

**Proposition 2.** *Given a state $\omega = (a, s)$, let $\bar{\phi}_A(\omega)$ denote the optimal fee rate for each of the a revenue maximizing activist funds and $\bar{\phi}_S(\omega)$ be the optimal fee rate for each of the s revenue maximizing stock selector funds. Under Assumption 1, the optimal managerial fees are:*

$$\begin{aligned}\bar{\phi}_A(\omega) =& \frac{(2(N-a)-s+1)(N(\delta_I(\omega)+\bar{\phi}_I-\delta_A)-s(\delta_S-\delta_A))}{(2N-s-a+1)(2(N-a-s)+1)} \\ &- \frac{s(N(\delta_I(\omega)+\bar{\phi}_I-\delta_S)-a(\delta_A-\delta_S))}{(2N-s-a+1)(2(N-a-s)+1)}, \end{aligned} \tag{25}$$

$$\begin{aligned}\bar{\phi}_S(\omega) =& \frac{(2(N-s)-a+1)(N(\delta_I(\omega)+\bar{\phi}_I-\delta_S)-a(\delta_A-\delta_S))}{(2N-s-a+1)(2(N-a-s)+1)} \\ &- \frac{a(N(\delta_I(\omega)+\bar{\phi}_I-\delta_A)-s(\delta_S-\delta_A))}{(2N-s-a+1)(2(N-a-s)+1)}. \end{aligned} \tag{26}$$

In the special case where activist and stock selector investments are equally efficient ($\delta_A = \delta_S$), these funds will optimally charge identical rates. In this case, equilibrium fee rates reduce to

$$\bar{\phi}_A(\omega) = \bar{\phi}_S(\omega) = \frac{N(\delta_I(\omega)+\bar{\phi}_I-\delta_A)}{2N-s-a+1}. \tag{27}$$

We focus on this case in our numerical examples.

## 4.2 The Household's Problem

The household's problem, in Equation (11), is essentially a conventional planner's problem with affine production technologies and no capital adjustment costs. The novelty is that the risk and productivity characteristics of the production technologies arise from equilibrium in the fund manager labor market.[9] We make the usual conjecture that the value function can be written

$$V(\omega, K) = \frac{1}{1-\gamma} H(\omega) K^{1-\gamma}, \tag{28}$$

for a function $H(\omega)$ to be determined.

The value function satisfies Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned} &\frac{1}{1-\gamma}[\rho H(\omega) - \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'} H(\omega')] \\ &= \max_{c,\pi_A,\pi_S} \left\{ u(c) + (\mu - c - \pi_A(\bar{\phi}_A(\omega) + \delta_A) - \pi_S(\bar{\phi}_S(\omega) + \delta_S) - (1 - \pi_A - \pi_S)(\bar{\phi}_I + \delta_I(\omega))H(\omega) \right. \\ &\quad \left. - \frac{\gamma H(\omega)}{2} \left[ \bar{\sigma}^2 + \frac{\sigma^2}{N} \left( 1 - 2\pi_A \pi_S + \left( \frac{N}{a} - 1 \right) \pi_A^2 + \left( \frac{N}{s} - 1 \right) \pi_S^2 \right) \right] \right\}. \end{aligned} \tag{29}$$

The solution to the asset allocation problem is similar to Merton (1969): it reflects the mean returns and covariance matrix for the funds. Because we assume frictionless capital allocation and time-additively-separable utility, state transitions do not introduce a hedging

[9] Our solution of the household's problem follows Sotomayor and Cadenillas (2009), who establish optimality conditions in a setting similar to Merton (1969), but with market opportunities and utility contingent upon a continuous-time Markov process. See in particular Theorem 3.2, which covers the case where utility is not directly contingent on the Markov state. Although we generalize to a production setting and endogenous investment opportunities, the household's problem remains very similar to Sotomayor and Cadenillas (2009).

component to the asset allocation problem. Optimal $c$, $\pi_A$, and $\pi_S$ are

$$c(\omega) = H(\omega)^{-1/\gamma}, \tag{30}$$

$$\pi_A(\omega) = \frac{a\left((N-s)(\delta_I(\omega)+\bar{\phi}_I-\bar{\phi}_A(\omega)-\delta_A)+s(\delta_I(\omega)+\bar{\phi}_I-\bar{\phi}_S(\omega)-\delta_S)\right)}{\gamma\sigma^2(N-a-s)}, \tag{31}$$

$$\pi_S(\omega) = \frac{s\left((N-a)(\delta_I(\omega)+\bar{\phi}_I-\bar{\phi}_S(\omega)-\delta_S)+a(\delta_I(\omega)+\bar{\phi}_I-\bar{\phi}_A(\omega)-\delta_A)\right)}{\gamma\sigma^2(N-a-s)}, \tag{32}$$

under Assumption 1, which requires $\pi_A(\omega)+\pi_S(\omega) < 1$. To rule out negative productive investment, short selling of the index is prohibited. Optimal portfolios and fees for states in which the index short sale constraint binds are given in Appendix B. Such states occur with low probability in our numerical examples.

To condense notation, write the drift and variance of the household's optimal portfolio returns, respectively, as

$$\begin{aligned}\hat{\mu}(\omega) = {} & \mu - \pi_A(\omega)(\bar{\phi}_A(\omega)+\delta_A) - \pi_S(\omega)(\bar{\phi}_S(\omega)+\delta_S) \\ & - (1-\pi_A(\omega)-\pi_S(\omega))(\bar{\phi}_I+\delta_I(\omega)),\end{aligned} \tag{33}$$

$$\hat{\sigma}(\omega) = \bar{\sigma} + \frac{\sigma^2}{N}\left(1 - 2\pi_A(\omega)\pi_S(\omega) + \left(\frac{N}{a}-1\right)\pi_A(\omega)^2 + \left(\frac{N}{s}-1\right)\pi_S(\omega)^2\right). \tag{34}$$

Substituting the optimal portfolio choice and consumption and making use of the condensed notation, the HJB equation is

$$\gamma H(\omega)^{(\gamma-1)/\gamma} + \left((1-\gamma)\left(\hat{\mu}(\omega)-\frac{\gamma\hat{\sigma}(\omega)}{2}\right)-\rho\right)H(\omega) + \sum_{\omega'\in\Omega}\lambda_{\omega,\omega'}H(\omega') = 0. \tag{35}$$

The risk-free rate under the household's pricing kernel is

$$r_f(\omega) = \rho + \gamma\left(\hat{\mu}(\omega)-c(\omega)\right) - \frac{1}{2}\gamma\left(1+\gamma\right)\hat{\sigma}^2(\omega). \tag{36}$$

# 5 Results

We characterize model behavior via a series of propositions and numerical examples. We begin with some analytical results conditional on the state of the managed fund market, $\omega = (a, s)$. These provide intuition for some mechanisms at work in the model, and inform our numerical experiments. We then characterize the dynamic model for a set of baseline parameters. Finally, we study the phenomenon of fee compression and related changes in the relative competitiveness of index funds, activists, and stock selectors.

## 5.1 Analytical Results Conditional on State

The results in this section condition on the state of fund market competition, $\omega = (a, s)$, and describe how the equilibrium changes on the transition to an adjacent state. We show that transitions to states with more selectors are beneficial to the remaining active fund managers (selectors and activists), because both fees and AUM per fund increase, thereby increasing fund revenue flows to managers. Such transitions may worsen conditions for households, however, as fund managers capture a larger share of the benefits from active management.

By contrast transitions to states with more activists lead to lower fees and higher average fund efficiency, benefiting households. This is because a fund management industry with many activists is disciplined by a highly efficient index fund. We also consider a reduction in the index fee, which benefits households conditional on the state remaining unchanged.

The results in this section explain the primary mechanisms driving behavior in our dynamic model. Although they typify model behavior, the following propositions do require another assumption. In addition to Assumption 1, we introduce:

**Assumption 2.** *Activists and selectors manage their capital with equal efficiency:* $\delta_A = \delta_S$.

Analytical results when $\delta_A \neq \delta_S$ can be found in Appendix C.

We begin with two propositions summarizing the effects of selector entry on fees and on the household's portfolio opportunity set.[10] Selectors may enter either by replacing an activist incumbent, or by matching with an industry that is only in the index. In the first case (Proposition 3) the number of actively managed funds remains the same, whereas in the second case (Proposition 4) the number of actively managed funds increases.

**Proposition 3.** *Consider a transition from state $\omega = (a, s)$ to state $\omega' = (a - \Delta s, s + \Delta s)$, for some integer $\Delta s > 0$, and $a + s < N$. This transition replaces $\Delta s$ activists with stock selectors. Under Assumptions 1 and 2, the transition increases activist fees, increases selector fees, decreases average industry efficiency ($\delta_I(\omega') > \delta_I(\omega)$), and worsens the household opportunity set.*

**Proposition 4.** *Consider a transition from state $\omega = (a, s)$ to state $\omega' = (a, s + \Delta s)$, for some integer $\Delta s > 0$ and $a + s + \Delta s < N$, representing the entry of one or more selectors. Under Assumptions 1 and 2, this transition increases activist fees, increases selector fees, and leaves index efficiency unchanged.*

Managed fund fees increase in both Proposition 3 and Proposition 4. However there are two different mechanisms at work. In Proposition 3 the number of activists decreases, so the index becomes less efficient as spillovers decrease. Since an efficient index disciplines managed fund fees, it is intuitive that such fees rise as index efficiency declines. In Proposition 4 index efficiency remains the same, and the total number of managed funds increases, which suggests a more competitive environment that could lower fees. However, managed funds are strategic complements: they become better diversified and less risky *as a group* when additional funds enter. This strategic complementarity drives the increase in managed fees.

The main difference between the two modes of selector entry is that the household's investment opportunity set worsens in Proposition 3, whereas the overall effect on the op-

[10] Since the household is, essentially, solving a mean-variance problem, we think of the household's 'portfolio opportunity set' as the efficient frontier.

portunity set is ambiguous in Proposition 4. Index efficiency remains the same and it is possible that increased diversification offsets the increase in fees for actively managed funds.

The next proposition considers activist entry. Recall that new activists always match with industries that are not currently part of a managed fund.

**Proposition 5.** *Consider a transition from state $\omega = (a, s)$ to state $\omega' = (a + \Delta a, s)$, for some integer $\Delta a > 0$ and $a + \Delta a + s < N$, representing the entry of one or more activists. Under Assumptions 1 and 2 and for sufficiently small $\bar{\phi}_I \geq 0$, this transition decreases activist fees, decreases selector fees, improves index efficiency, and therefore improves the household's opportunity set.*

Proposition 5 shows that activist entry has the opposite consequences of selector entry: fees decline and the investment opportunity set unambiguously improves. Overall these results highlight that competition from an efficient index disciplines actively managed fund fees, and index efficiency is tied to the number of activists in the market. Even though selectors run their investments with higher efficiency than the index, the entry of additional selectors may not benefit households, as actively managed funds capture more of the gains via increased fees, due to strategic complementarities.

We can also show that the entry of additional selectors raises AUM per actively managed fund, regardless of whether or not the new selectors replace activists.

**Proposition 6.** *Consider a transition from state $\omega = (a, s)$ to state $\omega' = (a, s + \Delta s)$, for some integer $\Delta s > 0$ and $a + s + \Delta s < N$, representing the entry of one or more selectors. Under Assumptions 1 and 2, this transition increases AUM per fund and total AUM invested with actively managed funds.*

**Proposition 7.** *Consider a transition from state $\omega = (a, s)$ to state $\omega' = (a - \Delta s, s + \Delta s)$, for some integer $\Delta s > 0$ and $a + s < N$. This transition replaces $\Delta s$ activists with stock* *selectors. Under Assumptions 1 and 2, this transition increases AUM per fund and total AUM invested with actively managed funds.*

In combination with Proposition 4 and Proposition 3, it follows that the entry of new selectors increases fee revenues for all remaining actively managed funds. It also follows that indexed AUM declines when additional selectors enter. When additional activists enter, the effect on AUM is ambiguous. This is due to the two previously mentioned mechanisms, increased index efficiency and strategic complementarity between actively managed funds, which work in opposite directions. The dominant mechanism depends on parameters and initial state.

Finally, we consider the index fee, which is obviously important for how active managers set their fees. In contrast to activist and selector fees, the passive index fee $\bar{\phi}_I$ is exogenous and constant. Although modeled in reduced form, we implicitly assume a competitive market for identical index funds, such that $\bar{\phi}_I$ represents the labor and capital costs required to run an index fund subject to the zero profit condition.[11] In keeping with this interpretation and conditional on the state, a reduction in $\bar{\phi}_I$ can be viewed as a form of increased index efficiency. The following proposition summarizes the effect of passive fees on activist and stock selector fees.

**Proposition 8.** *For any state $\omega$, decreasing the passive index fee $\bar{\phi}_I$ decreases both the activist fee $\bar{\phi}_A(\omega)$ and the stock selector fee $\bar{\phi}_S(\omega)$. Therefore a reduction in $\bar{\phi}_I$ improves the household's portfolio opportunity set conditional on state $\omega$.*

Proposition 8 suggests that a reduction in index fees $\bar{\phi}_I$ is conditionally welfare improving for households, in the sense that expected utility within a given state $\omega$ increases due to the improvement in the household's portfolio opportunity set. Although it is still possible that a reduction in $\bar{\phi}_I$ could unconditionally reduce household welfare, such a result must hinge on

[11] Two funds offering an identical index product would be sufficient to compete fees down to costs.

the dynamic effects of the fee reduction, i.e., on the incentives of activists and stock selectors to search in their respective labor markets. We present relevant examples in Section 6.1.

## 5.2 Characterization of the Equilibrium for Baseline Parameters

We characterize the dynamic equilibrium via a numerical example to complement the propositions in the previous section and to illustrate some of the key features of the model. These figures shed light on the trade offs present for potential and incumbent fund managers when the representative household makes an endogenous portfolio choice decision. Potential fund managers consider two key elements that vary with the model state: the present value of their fees if they successfully form a fund, and the probability per unit time of forming a fund. The present value of fees depend, in equilibrium, on the household's risk and return-based investment allocation across each fund type and the fee setting decision of incumbent funds.

Our baseline parameter values in Table 1 are chosen to be consistent with existing studies where possible, while approximating a few key statistics from the data. A few of the parameter choices deserve special attention. First, we assume that activist and selector funds are equally efficient, with $\delta_S = \delta_A = 0$. Therefore, they offer equivalent products, and charge identical fees. By contrast, inefficient industries have a mean depreciation of $\delta = 1.8\%$, and as a result, lower cash flows per unit of capital. Given the productivity of capital $\mu$ is 11%, the efficiency gain from monitoring is economically significant. Second, the number of industries, $N = 25$, is chosen for fast equilibrium computational time. Experiments with larger numbers of industries lead to similar results.

Finally, the search parameters, while consistent with the labor literature, are largely ad hoc in describing the labor market for asset managers.[12] We start with identical search

[12] Again, see Petrongolo and Pissarides (2001), Rogerson, Shimer, and Wright (2005), and Petrosky-Nadeau, Zhang, and Kuehn (2018) for examples.

| | Parameter | Value |
|---|---|---|
| Mean productivity of capital | $\mu$ | 0.11 |
| Inefficient deterministic cap. depreciation | $\delta$ | 0.018 |
| Activist deterministic cap. depreciation | $\delta_A$ | 0 |
| Stock selector deterministic cap. depreciation | $\delta_S$ | 0 |
| Firm-specific std. dev. of cap. depreciation | $\sigma$ | 0.3 |
| Systematic std. dev. of cap. depreciation | $\bar{\sigma}$ | 0.13 |
| Number of firms | $N$ | 25 |
| Household relative risk aversion | $\gamma$ | 4 |
| Household subjective discount rate | $\rho$ | 0.02 |
| Fund labor market share coefficient | $\nu$ | 0.5 |
| Stock selector matching coefficient | $\eta$ | 0.5 |
| Activist exogenous exit rate | $\theta_A$ | 0 |
| Stock selector exogenous exit rate | $\theta_S$ | 0.2 |
| Activist search cost | $\zeta_A$ | 0.001 |
| Stock selector search cost | $\zeta_S$ | 0.001 |
| Manager reservation utility coeff. | $\underline{c}$ | 0 |
| Passive index fee | $\bar{\phi}_I$ | 0 |

**Table 1: Parameter values.** The table reports the baseline parameter values used in our numerical examples.

parameters for both activists and stock selectors, and select parameters such that active fund AUM is just above 50% unconditionally, and about evenly split between selectors and activists. The index fee is $\bar{\phi}_I = 0$, whereas unconditional mean active fund fees are about 0.8%. Active and passive market shares and fees are roughly consistent with the current equity mutual fund market (Duval, 2021). Activists and selectors are not cleanly delineated in the data, so we target approximately equal market shares to support a range of comparative statics that might substantially increase or reduce those market shares.

Figure 1 illustrates the stationary distribution of activists and stock selectors under the Table 1 parameters. The top panels summarize the joint density, while the bottom panels summarize the marginal densities. In equilibrium, a non-degenerate distribution exists where the modal percentage of industries managed by activists is 44% and the modal percentage of industries managed by selectors is 32%. In separate experiments, we find that the stationary distribution for activists and stock selectors is non-degenerate across a wide range of reasonable parameter values. Given that fees are endogenous, it is optimal for activists and selectors to set fees such that they remain in the fund management sector.

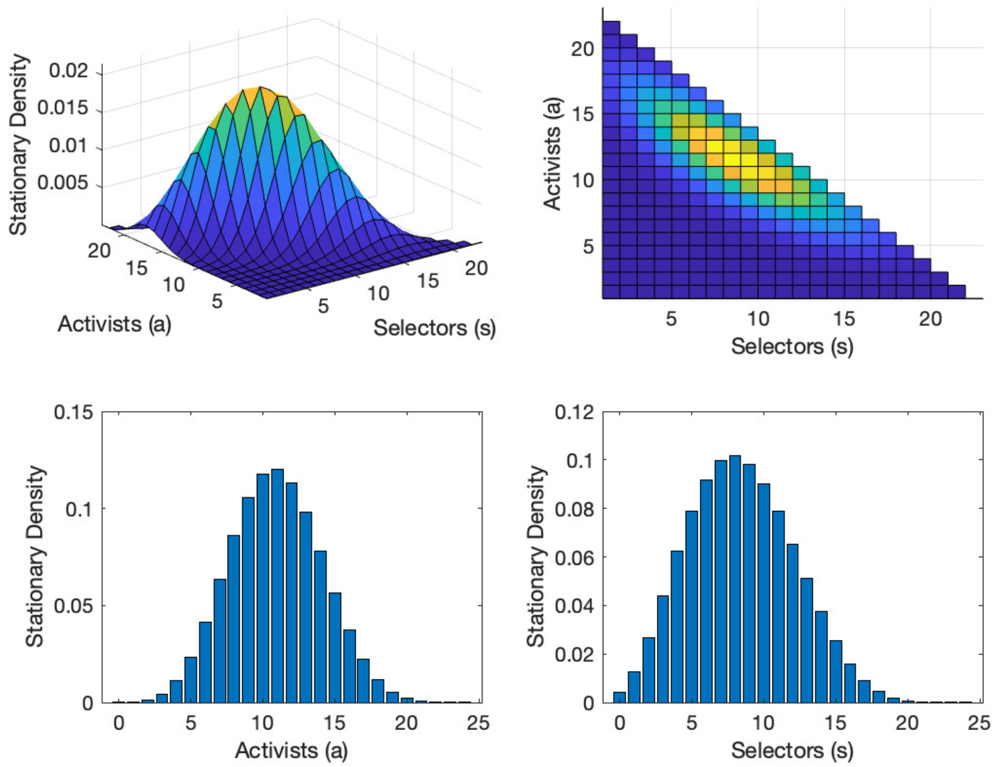


**Figure 1: Stationary Distribution of Activists and Selectors: Base Parameters.** The top left plot shows the joint stationary density of the number of selector ($s$) and activist ($a$) funds in the market, under example parameter values in Table 1. The top right plot represents the same information as a contour plot. The bottom plots show the marginal stationary densities of the number of activist ($a$) and selector ($s$) funds in the market.

While Figure 1 conveys information about the number of activists and stock selectors in equilibrium, Figure 2 demonstrates how the state of fund market competition impacts assets under management (AUM) through the household's portfolio problem. Understanding the demand for each fund type is crucial for understanding the dynamics of the active fund sector. The top row of the plots shows AUM as a fraction of wealth across the three fund types as a function of fund market competition, as summarized by the number of activists

($a$) and stock selectors ($s$). The bottom row shows plots illustrating different slices of the optimal AUM surface conditional on 1, 5, or 10 incumbent selector funds.

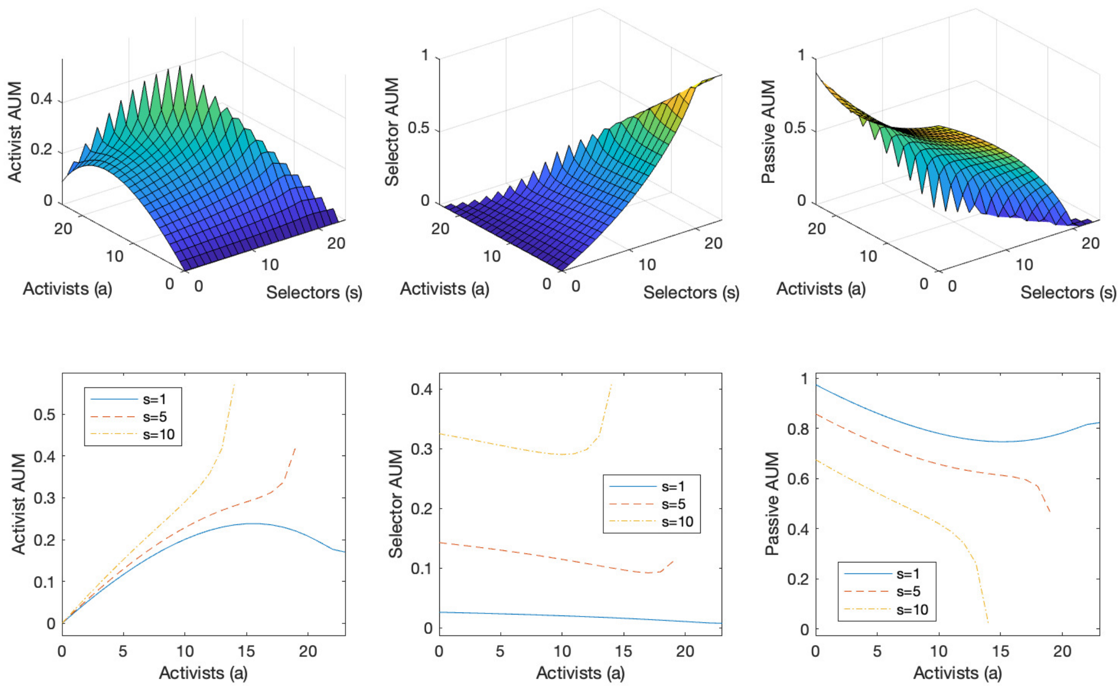


**Figure 2: AUM: Base Parameters.** The figure shows assets under management (AUM) in each of the three fund sectors, as a fraction of aggregate wealth, conditional on the state of competition in the fund manager market. Fund market competition is summarized by the tuple $(a, s)$, where $s$ is the number of selector funds, and $a$ the number of activist funds. The left plots summarize total activist AUM, the middle plots summarize total selector AUM, and the right plots summarize passive AUM. The top plots show three-dimensional surfaces, whereas the bottom plots show two-dimensional slices conditional on the number of selectors $s$. Parameter values are per Table 1.

Figure 2 is a good place to understand how competition among funds has diversification and spillover effects. These are offsetting effects in the case of activists, which is seen most clearly when there are few stock selectors, e.g., $s = 1$. Focusing on activist AUM when $s = 1$, as the number of activist funds in the market increases, the household will initially invest more in the active sector overall because more activists lead to better diversification with higher cash flows. However, the increase in the number of activist funds increases the number of efficient industries in the economy. This implies an increase in the industries' average cash flows (efficiency) in the passive index. This spillover effect eventually causes the household to shift its allocation towards the index fund, reducing the AUM among activists as more activists enter the market.

What happens when the number of activists is held constant, but the number of stock selectors increases, e.g., from $s = 1$ to $s = 5$ to $s = 10$? Consistent with Proposition 6, any slice through either the stock selector AUM surface or the activist AUM surface in the bottom plots of Figure 2 show that AUM is increasing in $s$. This is because the active management sector becomes better diversified, as each fund invests in a different industry with a different idiosyncratic shock, but since selectors generate no spillovers, the index fund does not become a more attractive investment alternative. The increase is larger when there are no — or very few — activist funds.[13]

Any funds not allocated to activists or stock selectors are invested in the passive index, so passive AUM is the mirror image of the aggregate active sector investment. The right column of Figure 2 shows that passive investment peaks when actively managed funds are few and poorly diversified or when many activists generate large spillover effects.

While Figure 2 provides information on the interplay between fund objectives and household portfolio choice, Figure 3 shows household welfare conditional on state $\omega = (a, s)$, normalizing the capital stock to unity, i.e., $\frac{1}{1-\gamma}H(\omega)$. The left panel shows that conditional welfare increases in the number of activists $a$. This is consistent with an improvement in the household's investment opportunity set when activists enter, per Proposition 5. However, the welfare measure in Figure 3 also incorporates probable future investment opportunities, based on equilibrium state transition rates from the current state. Essentially, Figure 3 shows that the intuition from Proposition 5 — that the entry of new activists is good news for households — is not somehow undone by the effect of the new activists on fund entry and exit dynamics.

[13]The plots for some activist and selector combinations are truncated when the corresponding states are unreachable. For example, for $s = 10$ stock selectors, the maximal number of activists is $a = 14$.

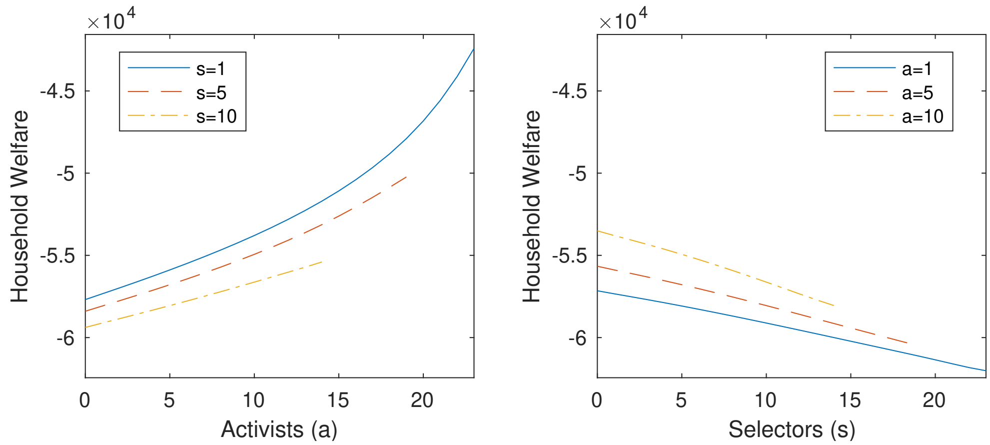


**Figure 3: Household Welfare: Base Parameters.** The figure shows properties of household welfare conditional on the state of competition in the fund market. Fund market competition is summarized by the tuple $(a, s)$, where $a$ is the number of activist funds, and $s$ the number of selector funds. The left plot shows household welfare relative to the number of activists $a$, while the right plot hows household welfare relative to the number of selectors $s$. Parameter values are per Table 1.

The right panel of Figure 3 shows household welfare is conditionally decreasing in the number of stock selectors, holding the number of activists constant. At first, this might seem surprising. After all, selectors run their investments as efficiently as activists, and with many selector funds in the market, it is possible for the household to assemble a diversified portfolio of stock selectors. This logic fails to consider two important aspects of the equilibrium though. First, an economy dominated by stock selectors is unlikely to transition to an economy dominated by activists and the positive spillovers that they offer. The second, and perhaps the more important reason that this simple intuition fails, is driven by optimal fee setting when stock selectors are prevalent.

The optimal fee policies are shown in Figure 4. The left column plots the fee rate, common for both activists and stock selectors, for different levels of fund competition. Equilibrium fees range from a few basis points to around 160 basis points. The top left panel shows the fee rate decreasing with the number of activists $a$, holding the number of selectors $s$ constant. Per Proposition 5, this pattern is characteristic of our model and holds for a broad range of parameter values, subject to some restrictions. The main mechanism is that the active management sector faces stiffer competition from the index when many activists generate spillovers.

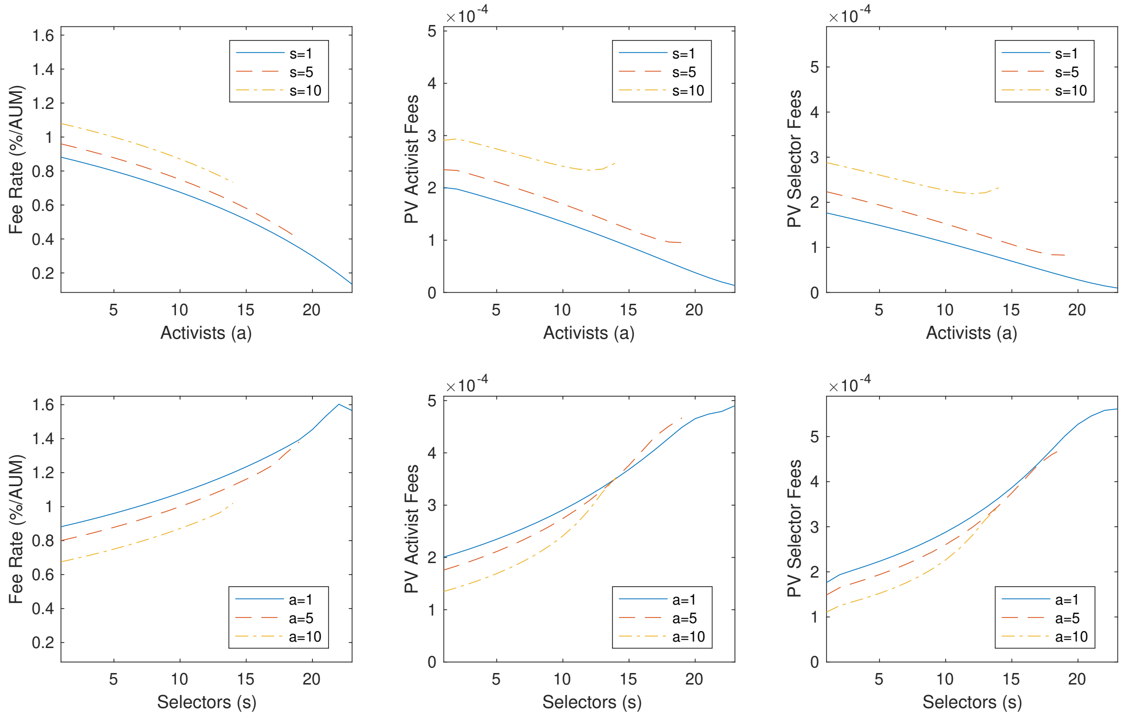


**Figure 4:** **Fees: Base Parameters.** The figure shows properties of fees conditional on the state of competition in the fund market. Fund market competition is summarized by the tuple $(a, s)$, where $a$ is the number of activist funds, and $s$ the number of selector funds. The left plots summarize the fee rate for both activists and selectors, the middle plots summarize the present value of activist fees per fund, and the right plots summarize the present value of selector fees per fund. The top plots are relative to the number of activists $a$, while the bottom plots are relative to the number of selectors $s$. Parameter values are per Table 1.

The bottom left panel of Figure 4 shows fees increasing in the number of stock selectors $s$, holding the number of activists constant at $a = 1$, $a = 5$, or $a = 10$. This is consistent with Proposition 4, and is not peculiar to our choice of parameters.[14] As previously discussed, the reason is that actively managed funds are strategic complements and since selectors do not

[14] Careful examination of Figure 4 shows that fees may decline in one case, when $a = 1$ and $s$ increases from 22 to 23. When there are many activists and few selectors, index investment may be zero, violating Assumption 1 necessary for Proposition 4 to hold.

generate spillovers, there is no offsetting increase in index efficiency. Therefore all actively managed funds are able to raise their fees.

To solidify the intuition behind these results, it is helpful to consider two limiting cases. First, consider a state in which all industries have selector funds, so $s = 25$, and $a = 0$.[15] An equally-weighed portfolio of stock selectors would have exactly the same risk as the index, but selector investments provide a higher cash flow per unit of capital since $\delta_S = 0$ whereas the index holds investments with mean depreciation $\delta = 1.8\%$. This condition holds because *none* of the industries are monitored by activists. Stock selectors would only need to charge fees slightly less than $\delta$ in order to drive passive investment to zero. For any individual stock selector, there is little benefit to fee cutting because the diversification motive of risk-averse investors limits the ability of these funds to increase market share through lower fees. As a result, equilibrium fees are high.

On the other hand, consider the limiting case in which all industries have activist funds, so $a = 25$, and $s = 0$. An equally-weighted portfolio of activists would have exactly the same risk as the index, and activists run their investments efficiently, with $\delta_A = 0$, but due to spillovers the index also offers mean depreciation $\delta_A = 0$. Therefore activists would have to charge less than the index fee to attract positive AUM, but the index fee in our example is zero. So, it is impossible for activists to earn positive fees at all in this scenario. These polar cases illustrate that the decreasing fees for activists in the top left plot and the increasing fees for selectors in the top right plot are driven by competition with the index investment alternative.

The left column of Figure 4 focused on state-contingent fees as a percentage of AUM. The middle and right columns examine the present value (PV) of all future fees, conditional on the model's state of competition. The PV of activist fees is in the middle column, and the PV of selector fees is in the right column. In general, present values follow the same

[15] In fact, we require $a+s < N$ in our numerical solution, so the states $s = 25$ and $a = 25$ are not reachable.

pattern as fee rates, increasing with the number of selectors and decreasing with the number of activists. However, there are counterexamples. These second-order effects depend on the state transition matrix and are not easily decomposed.

The other component of potential managers' search decisions is the state of industries in the market: potential activists match with inefficient industries exclusively in the index fund, whereas potential stock selectors match with either these industries or industries owned by incumbent activists. All else equal, the likelihood that an activist's search is successful is highest when there are few incumbent stock selectors or activists ($a + s$ is low), whereas the likelihood that a stock selector's search is successful is highest when there are few incumbent selectors ($s$ is low).

These considerations about search success interact with the present values of fees, shown in Figure 4, to determine search intensity, shown in Figure 5. For activists, search intensity is highest when there are few activists (top left) or a moderate number of stock selectors (bottom left). Intensity declines sharply as $a$ increases as the present value of fees falls. As $s$ increases, the search intensity is hump-shaped. While the present value of activist fees increases, the probability of a successful match also falls for high $s$ leading to a humped search intensity. For stock selectors, search intensity behavior is similar to activists with a few noticeable differences. First, selector search intensity also falls as the number of incumbent activists rises, but at a lower rate than that of potential activists as stock selectors can also match with incumbent activists. Second, like activists, selector search intensity is hump-shaped in the number of incumbent stock selectors but does not necessarily converge to zero for a high number of incumbent activists. As the fraction of industries with incumbent activists approaches one, potential stock selectors still have incentives to search to displace those activists.

Per Equation (2), search intensity and the availability of target industries determines the rate at which new funds enter and, in the case of activists, exit. In turn, this determines the

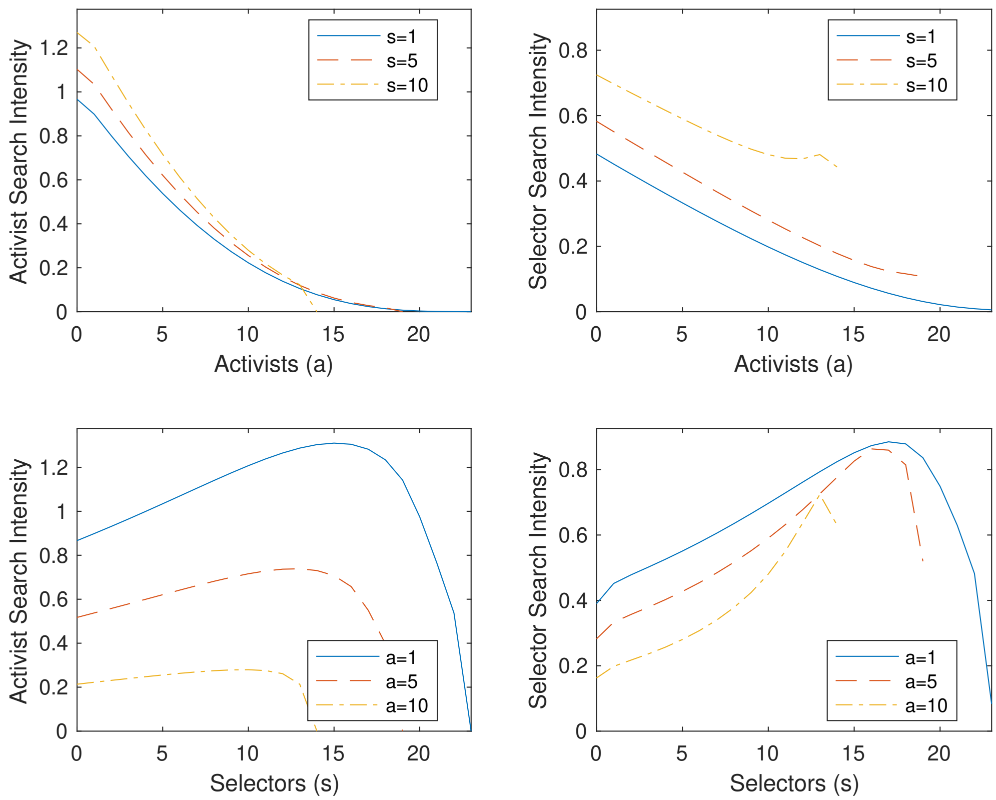


**Figure 5: Potential Manager Search Intensity: Base Parameters.** The figure shows the search intensity of potential activists (left plots) and potential selectors (right plots), conditional on the state of competition in the fund market. Fund market competition is summarized by the tuple $(a, s)$, where $a$ is the number of activist funds, and $s$ is the number of selector funds. The top plots are relative to the number of activists $a$, while the bottom plots are relative to the number of selectors $s$. Parameter values are per Table 1.

stationary distribution we began with in Figure 1.

# 6 Fee Compression and Fund Management Technologies

We can use the structure of the model to examine fee compression and its implications for households and the managed fund industry in general equilibrium. As a first step, we demonstrate the natural time-variation in fees in the dynamic equilibrium for our baseline parameters. Additionally, we consider changes in the relative competitiveness of stock selectors and activists through changes both search costs and investment efficiency (i.e., capital depreciation).

## 6.1 Fee Compression and Indexing

As documented in Duval (2020), the past two decades saw trends of increasing index share of AUM and declining fees for actively managed funds, even as the number of actively managed funds continued to grow. Between 2000 and 2019, average equity mutual fund fees declined from 1.06% to 0.74%, index share grew from 7.5% to 24.2% of long term mutual fund AUM, and the number of mutual funds increased from roughly 7,000 to over 21,000.[16] The phenomenon of decreasing fees in combination with increased competition and a shift towards passive management is referred to as fee compression.

Our dynamic model implies substantial variation in managed fund fees, even if there are no structural changes to the market. The top row of plots in Figure 6 shows the distributions of managed fund AUM (%) and the number of managed funds conditional on the fund fee. The bars represent the extremes of the distributions, while the lines represent the distribution averages. The bottom row of plots provides the probability density of fund fees and the average composition of the managed fund sector.

For our baseline parameters, fund fees range from about 0.1% to 1.6% of AUM, with most of the probability mass between 0.5% and 1%. The two top panels of Figure 6 show that the fractional AUM for managed funds declines as fund fees decline, while the average number of funds in the economy is non-monotonic in fees. This non-monotonicity is driven by significant changes in the composition of the managed fund sector. As fees fall, the mean

[16] The number of mutual funds is from the ICI Factbook, available online at https://www.icifactbook.org/ch2/20_fb_ch2, which includes all mutual funds. According to Duval (2020) there were 271 index funds in 2000 versus 492 index funds in 2019, implying that the number of funds excluding index funds has approximately tripled. However the number of actively managed funds is not separately stated, and the total includes some funds of funds that may not be considered active.

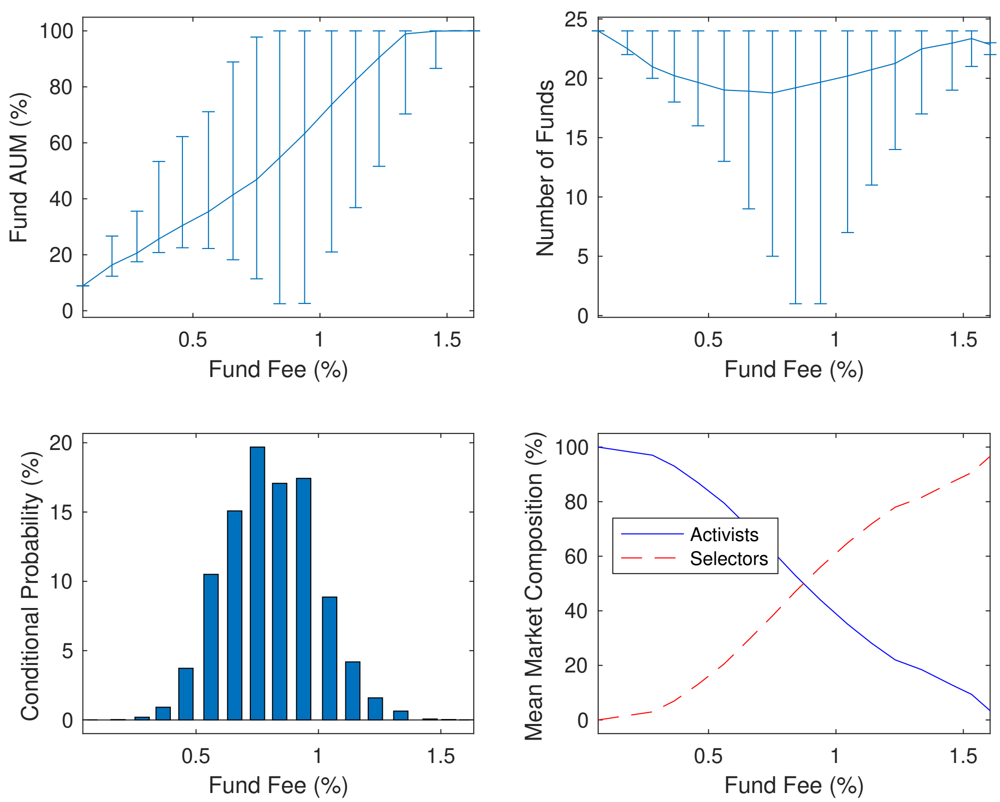


**Figure 6: Fund Market Competition and Fees: Base Parameters.** The figure illustrates the relationship between endogenous managed fund fees and the state of the managed fund market for baseline parameter values, per Table 1. In the top plots, the bars represent the extremes of the distributions, while the lines represent distribution averages. Activist and selector market composition, in the bottom right plot, is illustrated using each fund type's share of total active fund AUM.

composition of the managed fund sector switches from mainly stock selectors to mainly activists. This change in the composition of managed funds explains why fund AUM (%) is increasing with fees. When fees are low, it is because managed funds are dominated by activists, which make the passive index highly competitive due to spillovers. Managed funds lower their fees to compete with the low-fee index. Despite this competitive fee reduction by actively managed funds, the index is diversified and offers similar net-of-fees expected returns, so managed fund AUM is small. When fees are high, the managed fund market is dominated by stock selectors. Given selectors do not generate spillovers, the household has to invest a significant fraction of wealth in them to capture their benefit. The index charges a lower fee, but most of its investments are inefficient, so expected returns on the index are low. Managed funds can afford to raise their fees while still capturing a large percentage of AUM.

Structural changes and/or secular trends may be important features of the actual fund industry. However, the general pattern of falling fund fees, increasing numbers of funds, declining managed fund share of AUM and increasing passive fund share of AUM can be rationalized within our baseline dynamic model even without structural – or parameter – changes. As shown in the bottom right panel of Figure 6, the modal state is approximately 50% AUM for active management with fees of around 0.7%. From that modal state, a realized decline in fees of 1/4% and attendant increase in passive AUM of 15% or more with a simultaneous increase in number of managed funds is not improbable.

The results in Figure 6 assume passive index fees are constant. In the data, they have declined in recent years from an average of 0.27% in 2000 to 0.07% in 2019, according to Duval (2020). Since our model does not incorporate endogenous time-variation in index fees, we present a comparative static analysis of index fees, in Figure 7. Our results average over possible states, $\omega$, according to the stationary distribution implied by each set of model parameters. We vary index fees from 0% to 1.0%, a wider range than seen empirically, to illustrate the full range of model outcomes.

The bottom left panel of Figure 7 shows the relationship between the exogenous index fee (y-axis) and the endogenous mean actively managed fund fee, which we place on the x-axis for consistency with Figure 6. As the index fund fee increases from 0 to 1%, the mean active fund fee increases at a decreasing rate from around 0.8% to a maximum of almost 1.8%. In the upper left panel of Figure 7, the average AUM, as a percentage of the entire value of the capital stock, is an increasing and concave function of the mean fee. When index fees are zero and managed fund fees are at their minimum, the percentage of assets under active management is approximately 55%, and it increases to almost 100% when fees are at their

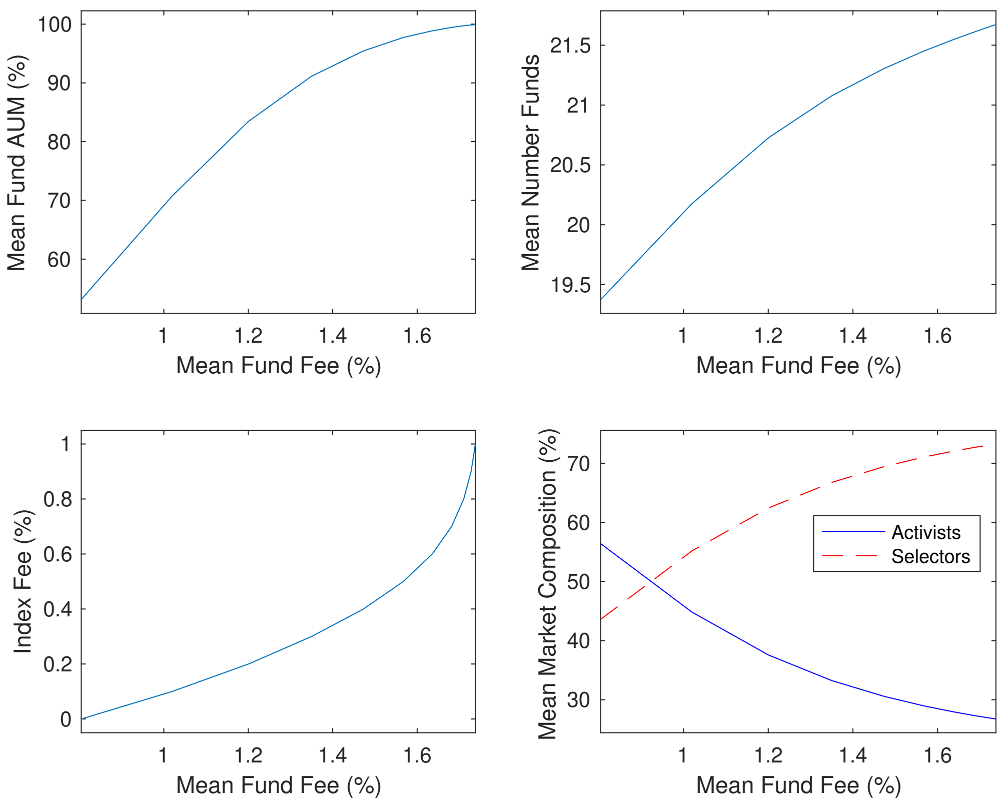


**Figure 7: Fund Market Competition and Index Fees.** The figure illustrates the relationship between mean managed fund fees and the state of the managed fund market for increasing passive index fee, $\bar{\pi}$, as shown in the bottom left plot. Remaining parameter values are per Table 1. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.

highest. The bottom right panel of Figure 7 shows that as mean active fund fees increase, the composition of the managed fund industry shifts from activists towards selectors. Similar to Figure 6, activists fare better in low fee scenarios than do stock selectors. The mean number of actively managed funds, in the upper right panel of Figure 7, rises from around 19.5 to around 21.5 as fees increase. Unlike in Figure 6, all relationships are monotonic.

It seems plausible, from the comparative static analysis in Figure 7, that a decline in index fees contributed to a decline in actively managed fund fees and to an increase in the passive index market share. Although we have not calibrated the model to match this moment, it is interesting that the mean impact of declining index fees on managed fund fees is of a similar scale to typical model dynamics without structural parameter changes, at least for the 0.2% decline in index fees observed since the year 2000.

So far the unconditional comparative static results from our dynamic model are consistent with the conditional results in Proposition 8: a decrease in the index fee decreases activist and selector fees. The right y-axis of the top right panel of Figure 8 shows that welfare implications are also consistent with Proposition 8: lower index fees increase household welfare unconditionally as well as conditionally, with parameters otherwise unchanged from Table 1.

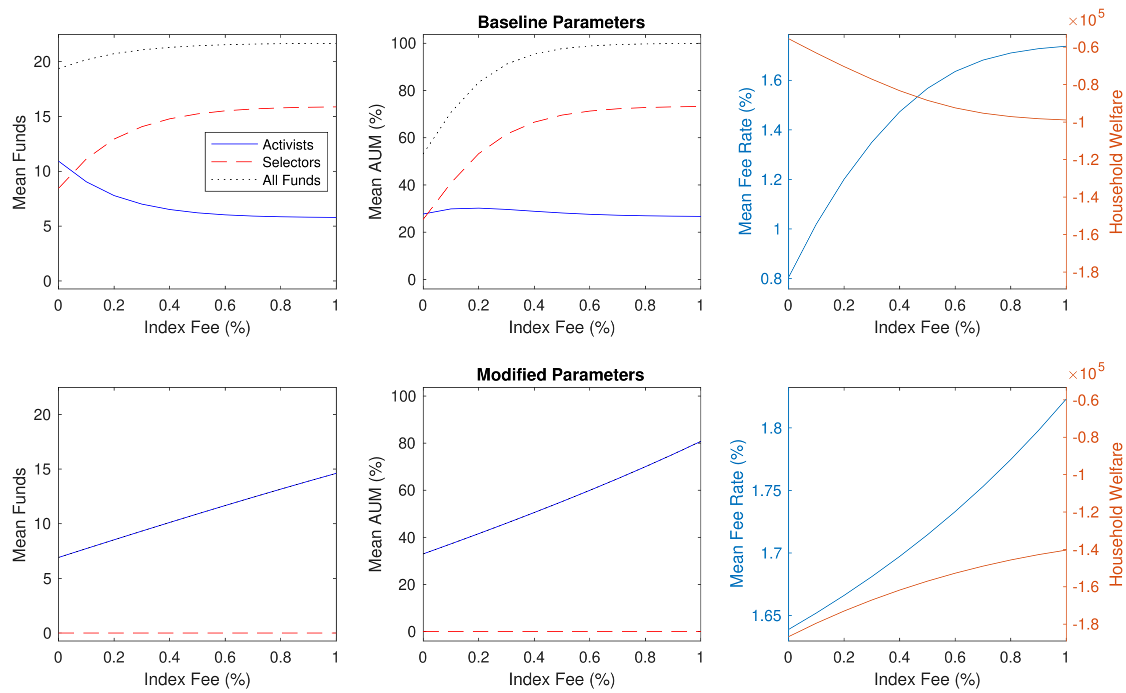


**Figure 8: Vary Index Fee.** The effect of increasing the passive index fee, $\bar{\pi}$, is illustrated in the above figure. In the top plots, all other parameters are per the baseline in Table 1. In the bottom plots, the baseline parameters are modified as follows: the fund manager's reservation utility $\underline{c} = 0.06\%$, the exogenous selector exit rate $\theta_S = 10$, the exogenous activist exit rate $\theta_A = 0.2$, and the unmonitored industry depreciation of $\delta = 4\%$. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.

Although higher index fees are typically bad for households, counterexamples exist. Low index fees make diversified investment inexpensive, but this crowds out active management, reducing average firm efficiency. If the reduction in efficiency is large enough relative to the risk reduction from improved diversification, household welfare declines. With our baseline parameters, the reduction in efficiency is mitigated because relatively more stock selectors are driven out of the market. The reduction in AUM from the shift towards passive investment reduces selector search intensity, which reduces the likelihood that an activist fund will be replaced by a selector fund. Therefore activist fund longevity increases as index fees decline. This mitigates the reduction in fee revenue per unit of time for activists, whereas for stock selectors there is no offsetting effect because selectors exit at a fixed exogenous rate.

As previously discussed in the context of Figure 3, households prefer activists to stock selectors from a welfare perspective, even when activist and selector investments are as efficiently run, because activists generate a spillover for the index while stock selectors do not. The relative shift towards activists is one reason the reduction in index fees benefits households on net.

The bottom panels of Figure 8 use alternative parameters in which stock selectors are shut down, and repeat the unconditional comparative static analysis with index fees ranging from 0% to 1%, using the stationary distributions corresponding to each set of model parameters.[17] In this context, low index fees are bad for households. The shift towards passive investment deters activists from entering the market, and the reduction in average firm efficiency is greater than the value of the fee reductions. In this way, the effects of cheap indexing on the incentive to engage in activism can overturn the intuition from Proposition 8, which holds fund market composition constant. *This conclusion is important and worth noting. Our analysis shows that the impact of fee compression on household welfare depends on the composition of the actively managed fund sector.*

[17] Baseline parameters in Table 1 are modified to have fund manager reservation utility $\underline{c} = 0.06\%$, exogenous selector exit rate $\theta_S = 10$, exogenous activist exit rate $\theta_A = 0.2$, and unmonitored industry depreciation of $\delta = 4\%$.

## 6.2 Changes to Fund Management Technologies

The previous section considered increased competition for active fund managers, in general, in the form of decreased index fees. However recent advances in computing technology, e.g., developments in machine learning combined with improved data aggregation and integration, may have enabled easier identification of investment prospects by active funds, potentially increasing competitiveness. Additionally, the Volcker Rule, enacted in 2014 as part of the Dodd-Frank Wall Street Reform and Consumer Protection Act, curtailed proprietary trading inside banks, impacting the labor market for active fund managers. We model these changes in reduced-form by varying the search costs for potential managers to form funds. We also model changes to the depreciation rates of activist- and selector-held firms to capture reduced-form changes in their monitoring or stock selection success which could also be driven by changes in technology.

Although our first motivating example of improved computing technology applies most naturally to stock selectors, active managers of all types make use of it to some extent. The Volcker Rule example potentially impacted the labor market for both types of managers. The effects of varying activist or selector search costs are shown in the top and bottom rows of Figure 9, respectively. We vary search costs from 0.0001 to 0.002, recalling that costs are 0.001 for activists and stock selectors in the baseline parameterization. The unconditional means, shown in the panels of the figure, average over possible states $\omega$ according to the stationary distribution implied by each set of model parameters All parameters, except search costs, are as in Table 1.

In most respects, the impact of reducing activist search costs is intuitively reasonable. More potential managers search for activist opportunities when costs are low, so the mean number of activists increases, and mean fees decline. Some of the increase in activist funds comes at the expense of selector funds, which decline in mean number with lower activist search costs. The total number of funds increases, so household welfare increases with lower

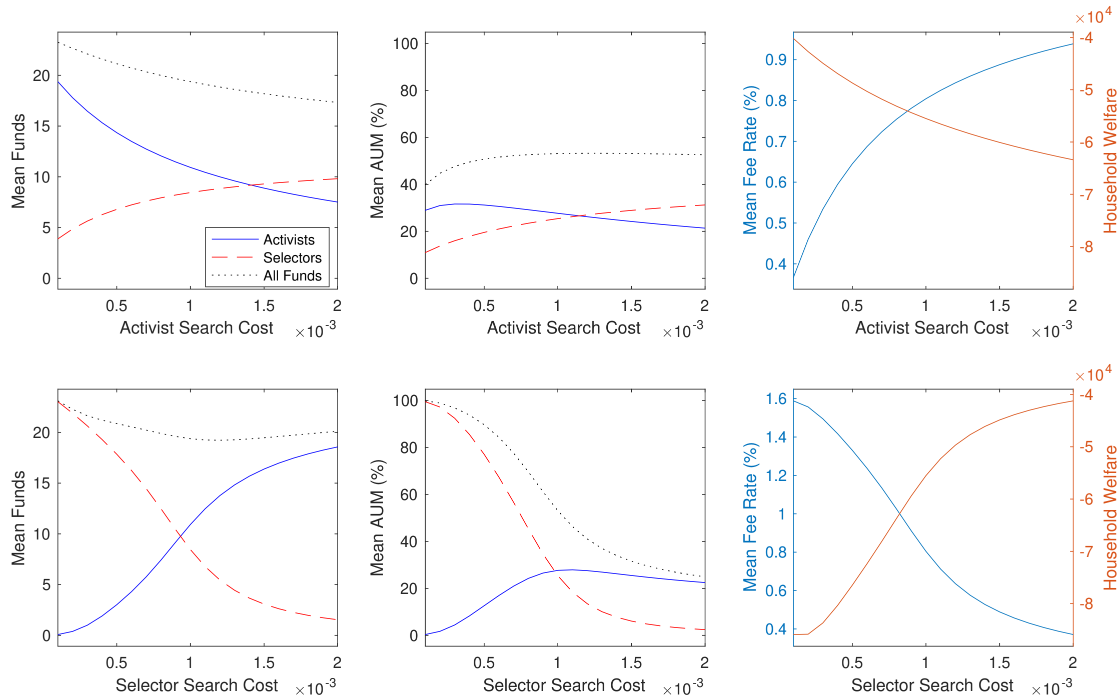


**Figure 9: Vary Search Costs** The figure shows the effects of varying search costs. The top plots vary activist search costs ($\zeta_A$), while the bottom plots vary selector search costs ($\zeta_S$). The remaining parameter values are per Table 1. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.

activist search costs, as industries are more efficiently run on average. The response of mean activist AUM to search costs is somewhat counterintuitive. Activists AUM initially increases as costs decline and more activists enter, but AUM declines for very low search costs, as the spillover effect from increased activism becomes so strong that households shift towards passive investment, around $\zeta_A = 0.0005$. Under this specific mechanism, decreased activist search costs may actually be bad for activists.

The effects of varying selector search costs are given in the bottom row of Figure 9. Analogous to the previous case, as selector search costs decline, both the average number of selectors and the AUM of selector funds increase. However, the number of activists declines rapidly given new stock selectors can displace them. These combined changes lead to increasing fund fees overall. This leads to a decrease in household welfare when selector search costs fall.

Selectors displace activists via two mechanisms in our model. The first is direct displacement, by forming funds in industries with activist incumbents, who are then forced out. This represents a form of predator-prey relationship between selectors and activists. Second, activists may be indirectly displaced by selectors simply because activists cannot enter industries with incumbent selectors. In Figure 10, we change the $\eta$ parameter from its baseline value of 0.5 to 1, shutting down the predator-prey relationship, and repeat the comparative static analysis of Figure 9 with otherwise identical parameters. Overall the results are similar, except that the mean number of activists is relatively unaffected by selector search costs until they decline below around $\zeta_Q = 0.0005$. Therefore the predator-prey relationship makes activists more sensitive to selector search intensity, but selectors would still displace activists even without this mechanism.

Over time it is also plausible that the incentives for activists to monitor the companies in which they invest have shifted, or that the effectiveness of stock selectors has changed. We model these changes, in reduced form, by varying the depreciation rate associated with activists or stock selectors. This alters the household's portfolio opportunity set directly, by changing the quality of individual selector or activist funds, in addition to altering the composition of the portfolio opportunity set, by changing the equilibrium composition of funds. Changes in the depreciation rate for activists capture changes in their ability to exercise voice, while changes in the depreciation rate for stock selectors capture changes in their ability to stock select successfully.

The effects of varying activist or stock selector depreciation are shown in the top and bottom rows of Figure 11, respectively. As in earlier figures, we show the unconditional means averaged over the stationary distribution implied by the model parameters in Table 1 and the relevant search cost parameters. Unlike the examples in Section 5.2, activist and selector fees differ in Figure 11 except when $\delta_A = \delta_S = 0$, matching our baseline parameter values. As

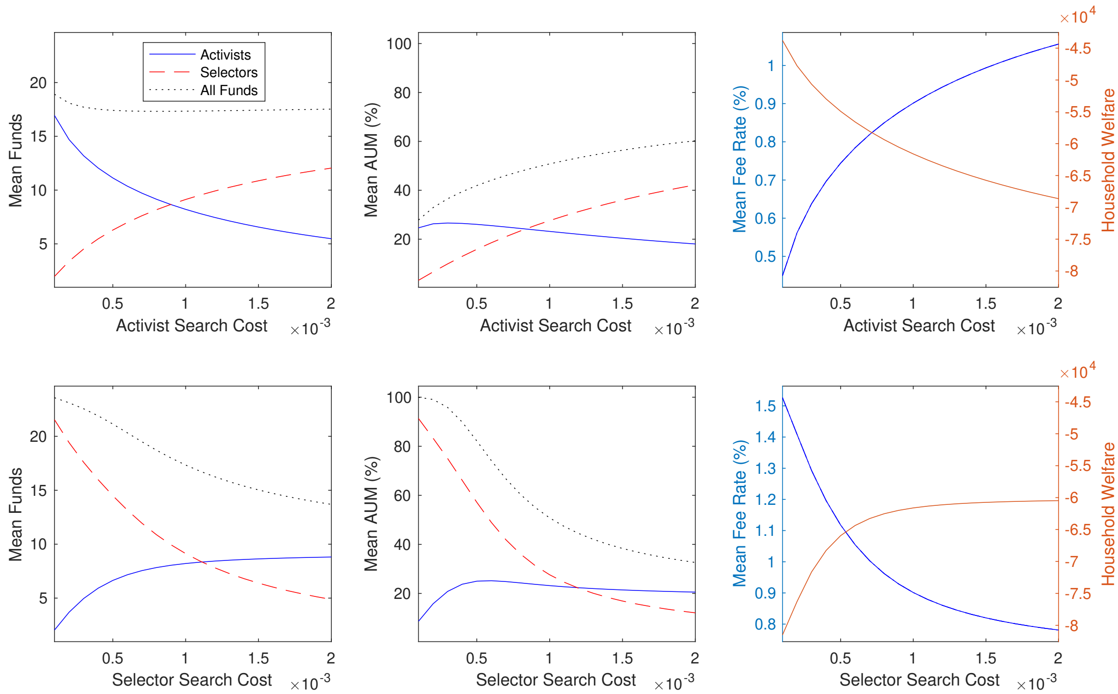


**Figure 10: Vary Search Costs: Modified Parameters** The figure shows the effects of varying search costs when $\eta = 1$, such that selectors enter by matching with industries present only in the index, and never directly displace activists. Remaining parameters are per Table 1. The top plots vary activist search costs ($\zeta_A$), while the bottom plots vary selector search costs ($\zeta_S$). Remaining parameter values are per Table 1. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.

activist depreciation declines towards zero, the number of activists increases slightly, activist AUM increases slightly, and activist fees are largely flat. The total number of funds does not change, so household welfare increases as the composition of the managed sector tilts towards activists. Activist gains in AUM come at the expense of the stock selectors because total fund AUM decreases slightly with lower activist depreciation. The biggest impact on stock selectors is a decline in fees, which fall from 0.9% to 0.8% as activist depreciation falls. The model shows that mean fund fees fall even though funds offer a more attractive product gross of fees, on average. *The overall impression is that the fund industry is healthier and more competitive when activists run their funds more efficiently.*

As shown in the bottom row of Figure 11, the effects of decreasing selector depreciation

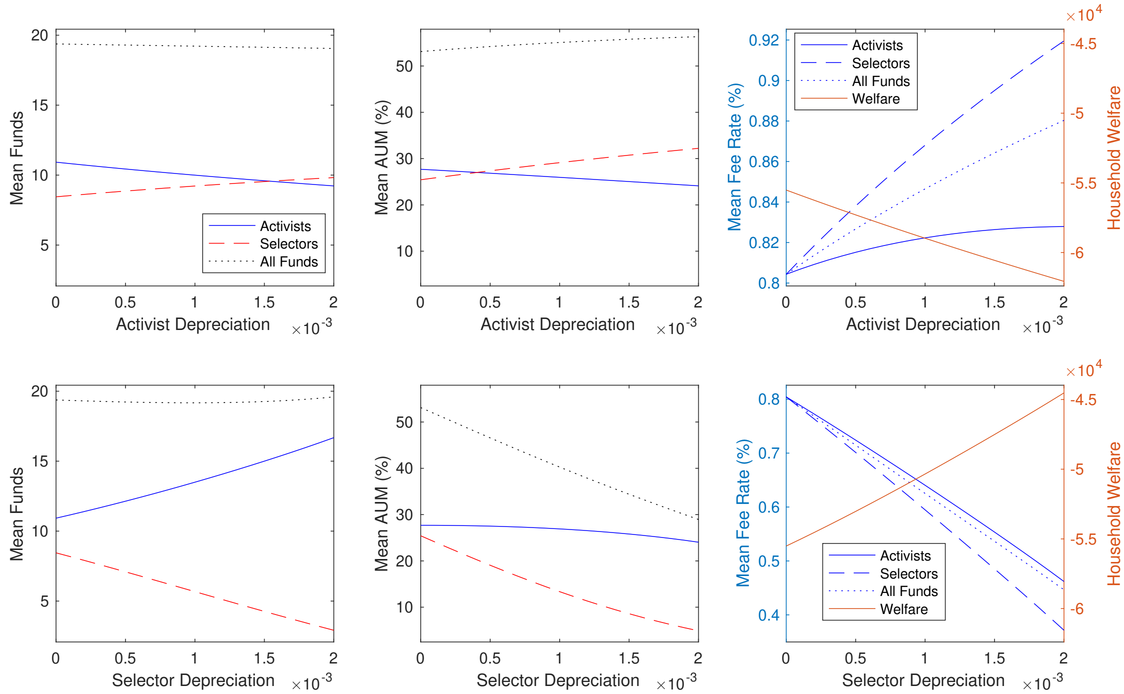


**Figure 11: Vary Depreciation (Efficiency).** The figure shows the effects of varying the capital depreciation rate, capturing how efficiently different firms are run. The top plots vary depreciation of firms run by activists ($\delta_A$), while the bottom plots vary depreciation of firms run by selectors ($\delta_S$). The remaining parameter values are per Table 1. Unconditional means are computed using the stationary distribution corresponding to each set of model parameters.

are stronger than the effects of decreasing activist depreciation. As selector depreciation approaches zero, the composition of the managed fund industry tilts toward fewer activist funds and more selector funds. While selector AUM increases as selector depreciation falls, it comes at the index fund's expense as activist AUM also increases slightly.

Households substitute into stock selectors and out of the index fund because they need to own the selector funds to capture the higher firm efficiency of selector-owned firms. Given that the cheaper index fund is less desirable, mean managed fund fees rise with lower selector depreciation leading to lower household welfare. *In the model, it is better for the fund industry than it is for households when stock selectors become more efficient.*

# 7 Conclusion

Active fund management has been transformed over the past two decades. The industry has seen significant changes in the costs of running a fund and increased competition from index products, delivered through both traditional open-end funds and exchange-traded funds. There has also been an increase in competition within the actively managed sector with refinements in strategies based on both voice and stock selection. All of this change has occurred under increasing calls for regulation of the industry and amid arguments in the political arena about both its practices and its size.

We construct a stylized general equilibrium model of the fund industry in order to better understand the implications for household welfare of some of these changes. In our model, activist managers exercise voice to improve the operating efficiency of the assets they own. Voice generates a positive spillover effect because passive index investments in overlapping industries also benefit from the efficiency improvements. Stock selectors are able to invest in a subset of the efficiently managed assets. However, because selectors do not monitor firms, their social benefits are significantly different from activist funds — *even though both fund types offer identical return distributions to households.* The model delivers endogenous fee structures, for both types of actively managed funds, and non-degenerate steady state distributions of the sizes of the two managed sectors. We use the model to conduct simple quantitative experiments that increase our understanding of the impact on the fund industry and household welfare of changes in search costs, managerial efficiency, and fee compression. We highlight interesting endogenous nonlinearities in the model equilibrium and provide some insight into the nature of the spillover effects created by active management.

We show that the distinction between voice and selection has important implications for the equilibrium size of the active management sector as a whole and for the share of benefits from active management that are passed through to households or retained within the fund industry. Although the model is simple enough to clearly delineate the important margins affecting the decisions of both managers and households, it is rich enough to deliver nontrivial implications for the impacts of fees and costs on the composition of the fund industry and household welfare. *The pivotal role of compositional changes within the fund industry for household welfare is, perhaps, the most important implication of our modeling framework.*

While activism and voice has received substantial attention in the empirical literature, it has not been as widely examined in theory. Our analysis is an attempt to begin that conversation. Active management based on fundamental analysis, as initiated by Graham and Dodd, has increasingly embraced voice as a complement to stock selection. Our model draws a bright line between these two activities to highlight the welfare effects of activism. We believe that a greater understanding of this complementarity is an important avenue for future research.

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# Appendix

## A Subdivision of Industries into Firms

In order to microfound certain assumptions about stock selector investment funds, we further subdivide each industry into a continuum of firms indexed $j \in \Omega_i$, where $\Omega_i$ is the set of firms in industry $i$, assumed to have positive mass. Capital invested in firm $j$ depreciates at rate

$$K_{j,t}\left[-\delta^i_{j,t}dt + \overline{\sigma}d\overline{W}_t + \sigma dW_{i,t} + dW_{j,t}\right], \tag{37}$$

where $\delta^i_{j,t}$ is depreciation drift of firm $j$ in industry $i$, and $W_{j,t}$ is an independent Brownian motion specific to firm $j$.[18]

We decompose the mean depreciation of each firm $j$ into industry and firm-specific components:

$$\delta^i_{j,t} = \delta_{i,t} + \delta_j. \tag{38}$$

The average of the firm-specific $\delta_j$ within each industry is zero; otherwise the details of its distribution are not conceptually important. For simplicity and without loss of generality, we assume that the firm-specific component $\delta_j$ is either $\delta_L < 0$ (low) or $\delta_H > 0$ (high), and that $\delta_H = -\delta_L$.

Each incumbent stock selector identifies the subset of firms within his industry $i$ with low depreciation $\delta_j = \delta_L$, and invests in only that subset of firms. In this way selectors perform security selection, modeled in reduced form. Because each stock selector invests in a positive mass of firms within its industry, idiosyncratic firm risk $W_{j,t}$ is diversified away, leaving only industry risk $W_{i,t}$ and systematic risk $\overline{W}_t$. We assume that index and activist investors cannot tell if a firm is of type $L$ or $H$, but they wish to eliminate firm-specific risk. Therefore they invest equally in all firms within each industry, with average $\delta_j = 0$.[19]

It follows that a selector fund invested in industry $i$ has depreciation (gross of fees)

$$K_{i,t}\left[-(\delta_{i,t} + \delta_L)dt + \overline{\sigma}d\overline{W}_t + \sigma dW_{i,t}\right], \tag{39}$$

whereas activist or index funds invested in industry $i$ have depreciation following Equation (1).

Suppose towards a contradiction that an activist fund invests in industry $i$ simultaneously with a stock selector. Then industry $i$ is efficient with $\delta_{i,t} = \delta_A$, and mean depreciation of the activist fund is $\delta_A$ (gross of fees). Meanwhile the selector fund in the same industry has mean depreciation $\delta_A + \delta_L < \delta_A$. Returns on the two funds are perfectly correlated, consisting only of industry risk $W_{i,t}$ and systematic risk $\overline{W}_t$. Activists and stock selectors are free to set their fees to maximize revenues, but for any positive activist fee, there exists a positive selector fee such that the selector fund has higher expected returns net of fees than

[18]The volatility of $W_{j,t}$ is ultimately irrelevant provided it is finite, so assuming a standard Brownian motion is without loss of generality.

[19]Equivalently we could assume that funds randomly sample a large number of firms within each industry.

the activist fund. Since the household can access both the activist fund and the selector fund in industry $i$ simultaneously, this presents an arbitrage opportunity in which the household goes long the stock selector and short the activist. Since we assume that funds exit if their AUM is less than or equal to zero, a stock selector investing in the same industry as an activist can always drive the activist's AUM to zero in order to capture all fee revenues in that industry.[20]

It follows that any incumbent activist exits when a stock selector enters the same industry, which further implies that any industry with an incumbent stock selector has equally-weighted average depreciation $\delta_{i,t} = \delta$. Effective selector depreciation is constant, and we write $\delta_S = \delta + \delta_L$.

## B Fees with a short sale constraint

Following convention, we assume that capital allocated to each productive technology must be non-negative, which in turn requires a short sale constraint on fund AUM. Because activist and selector funds exit unless AUM is positive, the short sale constraint only ever binds passive index investment. The index short sale constraint (SSC) affects household capital allocation across the remaining unconstrained funds, and the fee schedules of those fund managers.

There are three possible scenarios:

1. The household optimally chooses positive index investment. Activists and selectors set fees consistent with positive index investment.

2. The household wishes to short the index, and the SSC binds. Activists and selectors set fees consistent with no index investment, and the index SSC binds in equilibrium.

3. A boundary solution where the index SSC binds at the margin. Activists and selectors choose fees such that the household optimally chooses exactly zero index investment.

For case 1, the household's optimal portfolio is given by Equations (31) and (32), and optimal fees are per Proposition 2. This section describes equilibrium in cases 2 and 3.

[20]This ignores the possibility that, for example, the stock selector internalizes the activist's positive effect on industry productivity, and sets fees to match the activist's net of fee returns. Household asset allocation to the selector and activist funds within industry $i$ would be indeterminate in this case, so it is difficult to say if such a scenario would ever be to the stock selector's advantage. Since the objective is merely to provide some plausible microfoundation for our assumptions, we ignore this and other esoteric scenarios.

## B.1 Interior solution with binding index short sale constraint

In case 2, the index SSC binds, and the representative household effectively solves a two-sector portfolio optimization problem, with solution

$$\pi_A(\omega) = \frac{as\left(\bar{\phi}_S(\omega) + \delta_S - \bar{\phi}_A(\omega) - \delta_A\right) + a\gamma\sigma^2}{(a+s)\gamma\sigma^2}, \tag{40}$$

$$\pi_S(\omega) = 1 - \pi_A(\omega). \tag{41}$$

The Nash equilibrium in fees is

$$\bar{\phi}_A(\omega) = \frac{s(\delta_S - \delta_A)}{2a + 2s - 1} + \frac{\gamma\sigma^2}{a+s-1}, \tag{42}$$

$$\bar{\phi}_S(\omega) = \frac{a(\delta_A - \delta_S)}{2a + 2s - 1} + \frac{\gamma\sigma^2}{a+s-1}, \tag{43}$$

or if $\delta_A = \delta_S$ then

$$\bar{\phi}_A(\omega) = \bar{\phi}_S(\omega) = \frac{\gamma\sigma^2}{a+s-1}. \tag{44}$$

## B.2 Boundary solution with index short sale constraint binding at the margin

In case 3, funds face a discontinuous objective in AUM in the neighborhood where the index SSC begins to bind. As with case 1 and 2, equilibria are pairs of fees for activists and selectors such that no individual fund manager can increase his fee revenues by increasing or decreasing his fees. However the solution is typically not unique: due to the discontinuous household response to fee changes, there is a continuum of fee pairs such that index AUM is exactly zero and individual fund managers can do no better by decreasing or increasing fees.

Therefore the challenge in case 3 is to select a particular equilibrium. In addition to fund managers' individual optimality conditions, all candidate fee pairs $(\bar{\phi}_A(\omega), \bar{\phi}_S(\omega))$ must satisfy

$$\bar{\phi}_A(\omega) = \bar{\phi}_I + \delta_I(\omega) - \delta_A + \frac{sN(\bar{\phi}_I + \delta_I(\omega) - \bar{\phi}_S(\omega) - \delta_S) - \gamma\sigma^2(N-a-s)}{aN}, \tag{45}$$

which is the fee condition that sets index investment to exactly zero in the unrestricted securities market. Conditional on a case 3 equilibrium we have one free variable, i.e., given $\bar{\phi}_S(\omega)$, Equation (45) yields $\bar{\phi}_A(\omega)$.

In the general case $\delta_A \neq \delta_S$, we numerically evaluate a grid of $\bar{\phi}_S(\omega)$ and corresponding $\bar{\phi}_A(\omega)$ such that individual optimality conditions are satisfied, and select the fee pair that maximizes joint revenues $\pi_A(\omega)\bar{\phi}_A(\omega) + \pi_S(\omega)\bar{\phi}_S(\omega)$. Implicitly, funds collude in the selection of sectoral fees when a variety of fee pairs is sustainable as individually optimal once established.[21]

[21] We do not microfound such collusion in the context of the current model, but a large theoretical literature in dynamic games considers potential mechanisms. See for example Athey, Bagwell, and Sanchirico (2004).

With identical depreciation $\delta_A = \delta_S$, for example with our baseline parameters, the joint revenue maximization criterion implies identical fees for activists and selectors:

$$\bar{\phi}_A(\omega) = \bar{\phi}_S(\omega) = \bar{\phi}_I + \frac{(N-a)(\delta - \delta_A)}{N} - \frac{\gamma\sigma^2(N-s-a)}{(s+a)N}. \tag{46}$$

# C Conditional comparative statics related to activist and selector efficiency, $\delta_A$ and $\delta_S$

A simplistic but perhaps seductive line of reasoning goes as follows: if stock selectors charge lower fees than activists, then an increase in the number of selector funds might benefit households by driving down overall fees in the market. This section briefly establishes that this line of reasoning does not hold in our model, and instead implications are roughly the opposite.

The results in this section condition on the state of fund market competition, $\omega = (a, s)$. We establish some properties of fees when stock selectors run their investments less efficiently than activists, i.e., $\delta_S > \delta_A$. We show that this is a necessary condition for selectors to undercut activist fees in our model.

**Proposition 9.** *For any state $\omega$ such that $a + s < N$, stock selectors charge lower fees than activists, $\bar{\phi}_S(\omega) < \bar{\phi}_A(\omega)$, if and only if selector investments have higher depreciation than activist investments, $\delta_S > \delta_A$.*

Essentially, it is not optimal for a stock selector to charge a lower fee than an activist unless the stock selector's product is inferior.

The following result relates efficiency, fees, and expected returns conditional on the state $\omega$.

**Proposition 10.** *Suppose stock selector firms are less efficient than activist firms, $\delta_S > \delta_A$. Then for any state $\omega$ such that $a + s < N$, selector funds have lower net expected returns than activist funds: $\mu - \delta_S - \bar{\phi}_S(\omega) < \mu - \delta_A - \bar{\phi}_A(\omega)$.*

Although Proposition 10 conditions on our choice of parameters $\delta_A$ and $\delta_S$, it would be equivalent to condition on relative fees, per Proposition 9. In combination, these propositions imply that any discount in fees charged by stock selectors relative to activists does not fully offset the relative inefficiency of selectors' investments. It could be detrimental to households if "low cost" selectors replaced activists, even if they charge lower fees, because expected returns for selectors net of fees are still lower than for activists.

The following proposition formalizes and also generalizes this intuition. It is very similar to Proposition 3, but it relaxes Assumption 2.

**Proposition 11.** *Consider a transition from state $\omega = (a, s)$ to state $\omega' = (a - \Delta s, s + \Delta s)$, for some integer $\Delta s > 0$, and $a + s < N$. This transition replaces $\Delta s$ activists with stock selectors. The transition increases activist fees, increases selector fees, and decreases average efficiency: $\bar{\phi}_A(\omega') > \bar{\phi}_A(\omega)$, $\bar{\phi}_S(\omega') > \bar{\phi}_S(\omega)$, and $\delta_I(\omega') > \delta_I(\omega)$. This result holds for any $\delta_S < \delta$ and $\delta_A < \delta$.*

If selectors are less efficient than activists, then Proposition 11 implies that the household's portfolio opportunity set worsens when stock selectors replace activists. Although this result may seem obvious in light of the selectors' lower cash flows per unit of capital, it is less intuitive when equilibrium fees are considered. Proposition 9 shows that stock selectors charge lower fees than activists if $\delta_S > \delta_A$, but in equilibrium, transitioning to a state in which selectors replace activists increases fees for both types, and makes the household worse off.

However Proposition 11 shows that fees increase when selectors replace activists regardless of whether selectors run their investments more or less efficiently than activists. This highlights that spillovers from activism benefit households in two ways: directly by increasing index efficiency, and indirectly by disciplining fees.

# D Proofs

*Proof of Proposition 1.* We use a generalized Feynman-Kac theorem to express the value of the stochastic integral above as the solution to a PDE, which reduces to a simple system of equations in the Markov state variable due to homogeneity of the valuation in $K$.[22] Let $\mathcal{A}$ be the infinitesimal generator for $(\omega_t, K_t)$. Then the function $\Phi_A$ satisfies

$$\mathcal{A}\Phi_A(\omega, K) - (\beta_A(\omega) + r(\omega))\Phi_A(\omega, K) + \frac{\bar{\phi}_A(\omega)\pi_A(\omega)}{a}K = 0. \tag{47}$$

Recall our supposition $\Phi_A(\omega, K) = \phi_A(\omega)K$. Note that $\frac{\partial^2\Phi_A}{\partial K \partial K} = 0$ and the drift of $dK_t$ is $(r(\omega_t) - c(\omega_t)K_t$ under the risk-neutral measure, where $c(\omega_t)$ is the aggregate consumption-capital ratio, interpretable as a dividend yield. Following equation (2.9) in Zhu, Yin, and Baran (2015), we have

$$\mathcal{A}\Phi_A(\omega, K) = \phi_A(\omega)(r(\omega) - c(\omega))K + \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\phi_A(\omega') - \phi_A(\omega)]K. \tag{48}$$

The optimal $\phi_A(\omega)$ solves

$$\begin{aligned} &\phi_A(\omega)(r(\omega) - c(\omega))K + \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\phi_A(\omega') - \phi_A(\omega)]K \\ &\quad - (\beta_A(\omega) + r(\omega))\phi_A(\omega)K + \frac{\bar{\phi}_A(\omega)\pi_A(\omega)}{a}K = 0, \end{aligned} \tag{49}$$

$$\Rightarrow \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\phi_A(\omega') - \phi_A(\omega)] - (\beta_A(\omega) + c(\omega))\phi_A(\omega) + \frac{\bar{\phi}_A(\omega)\pi_A(\omega)}{a} = 0. \tag{50}$$

[22] See Zhu, Yin, and Baran (2015), Theorem 3.2.

Similarly, the function $\Phi_S$ satisfies

$$\mathcal{A}\Phi_S(\omega, K) - (\beta_S(\omega) + r(\omega))\Phi_S(\omega, K) + \frac{\bar{\phi}_S(\omega)\pi_S(\omega)}{s}K = 0, \tag{51}$$

where

$$\mathcal{A}\Phi_S(\omega, K) = \phi_S(\omega)(r(\omega) - c(\omega))K + \sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\phi_S(\omega') - \phi_S(\omega)]K. \tag{52}$$

After simplification this leaves

$$\sum_{\omega' \in \Omega} \lambda_{\omega,\omega'}[\phi_S(\omega') - \phi_S(\omega)] - (\beta_S(\omega) + c(\omega))\phi_S(\omega) + \frac{\bar{\phi}_S(\omega)\pi_S(\omega)}{s} = 0. \tag{53}$$

Subject to $\phi_A(\omega)$ and $\phi_S(\omega)$ satisfying Equation (50) and Equation (53), respectively, managerial search intensities $\hat{a}$ and $\hat{s}$ satisfy Equation (6) and Equation (7), respectively.

Therefore instantaneous transition rates are

$$\lambda_{(a,s),(a+1,s)} = \hat{a}^{1-\nu}(N - a - s)^{\nu}, \tag{54}$$

$$\lambda_{(a,s),(a-1,s+1)} = (1 - \eta)a \left(\frac{\hat{s}}{\eta(N - a - s) + (1 - \eta)a}\right)^{1-\nu}, \tag{55}$$

$$\lambda_{(a,s),(a,s+1)} = \eta(N - a - s) \left(\frac{\hat{s}}{\eta(N - a - s) + (1 - \eta)a}\right)^{1-\nu}, \tag{56}$$

$$\lambda_{(a,s),(a-s,s)} = \theta_A a, \tag{57}$$

$$\lambda_{(a,s),(a,s-1)} = \theta_S s, \tag{58}$$

and zero to all other states, subject to the additional restriction that transition intensity to states outside of $\{0 \ldots N\}$ is always zero. □

*Proof of Proposition 2.* For some state $\omega = (a, s)$ with $a > 0$, consider an individual activist who chooses his fee rate $\bar{\phi}'_A(\omega)$, while the remaining $a - 1$ activists charge fee $\bar{\phi}_A(\omega)$ and $s$ stock selectors charge fee $\bar{\phi}_S(\omega)$. Given his choice of fee, the individual activist will attract capital

$$\pi'_A(\omega) = \frac{N(\delta_I(\omega) + \bar{\phi}_I) - (N - s - a + 1)(\bar{\phi}'_A(\omega) + \delta_A) - s(\bar{\phi}_S(\omega) + \delta_S) - (a - 1)(\bar{\phi}_A(\omega) + \delta_A)}{\gamma\sigma^2(N - a - s)}. \tag{59}$$

The individual activist solves revenue maximization problem

$$\max_{\bar{\phi}'_A(\omega)} \bar{\phi}'_A(\omega)\pi'_A(\omega), \tag{60}$$

which has solution

$$\bar{\phi}'_A(\omega) = \frac{N(\delta_I(\omega) + \bar{\phi}_I - \delta_A) - s(\bar{\phi}_S(\omega) + \delta_S - \delta_A) - (a-1)\bar{\phi}_A(\omega)}{2(N - s - a + 1)}. \tag{61}$$

Similarly, for some state $\omega = (a, s)$ with $s > 0$, an individual stock selector choosing fee rate $\bar{\phi}'_S(\omega)$, while $a$ activists charge fee $\bar{\phi}_A(\omega)$ and the remaining $s-1$ stock selectors charge fee $\bar{\phi}_S(\omega)$, attracts capital

$$\pi_S{}'(\omega) = \frac{N(\delta_I(\omega) + \bar{\phi}_I) - (N - a - s + 1)(\bar{\phi}'_S(\omega) + \delta_S) - a(\bar{\phi}_A(\omega) + \delta_A) - (s-1)(\bar{\phi}_S(\omega) + \delta_S)}{\gamma\sigma^2(N - a - s)}. \tag{62}$$

The individual stock selector solves revenue maximization problem

$$\max_{\bar{\phi}'_S(\omega)} \bar{\phi}'_S(\omega)\pi'_S(\omega), \tag{63}$$

which has solution

$$\bar{\phi}'_S(\omega) = \frac{N(\delta_I(\omega) + \bar{\phi}_I - \delta_S) - a(\bar{\phi}_A(\omega) + \delta_A - \delta_S) - (s-1)\bar{\phi}_S(\omega)}{2(N - a - s + 1)}. \tag{64}$$

The symmetric Nash equilibrium is one in which all activists choose identical state-contingent fee $\bar{\phi}_A(\omega)$, all stock selectors choose identical state-contingent fee $\bar{\phi}_S(\omega)$, and no individual fund manager has an incentive to deviate from the rate schedule. These are the fixed points of the solutions to the respective revenue maximization problems, given by rate schedule

$$\begin{aligned}\bar{\phi}_A(\omega) =& \frac{(2(N-a) - s + 1)(N(\delta_I(\omega) + \bar{\phi}_I - \delta_A) - s(\delta_S - \delta_A))}{(2N - s - a + 1)(2(N - a - s) + 1)} \\ &- \frac{s(N(\delta_I(\omega) + \bar{\phi}_I - \delta_S) - a(\delta_A - \delta_S))}{(2N - s - a + 1)(2(N - s - a) + 1)},\end{aligned} \tag{65}$$

$$\begin{aligned}\bar{\phi}_S(\omega) =& \frac{(2(N-s) - a + 1)(N(\delta_I(\omega) + \bar{\phi}_I - \delta_S) - a(\delta_A - \delta_S))}{(2N - s - a + 1)(2(N - s - a) + 1)} \\ &- \frac{a(N(\delta_I(\omega) + \bar{\phi}_I - \delta_A) - s(\delta_S - \delta_A))}{(2N - s - a + 1)(2(N - a - s) + 1)}.\end{aligned} \tag{66}$$

□

*Proof of Proposition 3.* Since $\delta_A < \delta$ and $\delta_S < \delta$ is a standing assumption, the more general Proposition 11 applies. Therefore activist fees increase, selector fees increase, and index efficiency decreases whenever selectors replace activists. Under the additional assumption that $\delta_S = \delta_A$, the household's opportunity set worsens because opportunities for diversification remain the same, but net-of-fee expected returns are strictly worse. □

*Proof of Proposition 4.* Because only activists produce spillovers and the number of activists is unchanged by the transition, $\delta_I(\omega) = \delta_I(\omega')$.

Recall that when $\delta_A = \delta_S$ fees reduce to

$$\bar{\phi}_A(\omega) = \bar{\phi}_S(\omega) = \frac{N(\delta_I(\omega) + \bar{\phi}_I - \delta_A)}{2N - a - s + 1}. \tag{67}$$

Then the difference in fees, identical for activists or selectors, is

$$\bar{\phi}_A(\omega') - \bar{\phi}_A(\omega) \tag{68}$$

$$= \frac{N(\delta_I(\omega') + \bar{\phi}_I - \delta_A)}{2N - s - \Delta s - a + 1} - \frac{N(\delta_I(\omega) + \bar{\phi}_I - \delta_A)}{2N - a - s + 1} \tag{69}$$

$$= N(\delta_I(\omega) + \bar{\phi}_I - \delta_A)\left(\frac{\Delta s}{(2N - s - \Delta s - a + 1)(2N - a - s + 1)}\right) > 0. \tag{70}$$

□

*Proof of Proposition 5.* Note that for this transition

$$N(\delta_I(\omega') - \delta_I(\omega)) = -\Delta a(\delta - \delta_A). \tag{71}$$

Assuming the result and proceeding to show that it holds, we have

$$\bar{\phi}_A(\omega') - \bar{\phi}_A(\omega) < 0, \tag{72}$$

$$(2N - a - s + 1)N(\delta_I(\omega') + \bar{\phi}_I - \delta_A) - (2N - \Delta a - a - s + 1)N(\delta_I(\omega) + \bar{\phi}_I - \delta_A) < 0, \tag{73}$$

$$(2N - a - s + 1)N(\delta_I(\omega') - \delta_I(\omega)) + \Delta a N(\delta_I(\omega) + \bar{\phi}_I - \delta_A) < 0, \tag{74}$$

$$-(2N - a - s + 1)(\delta - \delta_A) + N(\delta_I(\omega) + \bar{\phi}_I - \delta_A) < -N(\delta - \delta_A) + N(\delta_I(\omega) + \bar{\phi}_I - \delta_A) < 0, \tag{75}$$

where the last step holds for sufficiently small $\bar{\phi}_I \geq 0$. □

*Proof of Proposition 6.* A few precursor results are necessary to prove the result.

Under Assumption 2, per-fund AUM, identical for activists and selectors, is

$$\frac{\pi_A(\omega)}{a} = \frac{\pi_S(\omega)}{s} = \frac{N(\delta_I(\omega) + \bar{\phi}_I - \bar{\phi}_A(\omega) - \delta_A)}{\gamma\sigma(N - a - s)}. \tag{76}$$

The difference in fees between the initial and destination states, identical for activists or selectors, is

$$\bar{\phi}_A(\omega') - \bar{\phi}_A(\omega) = \frac{\Delta s N(\delta_I(\omega) + \bar{\phi}_I - \delta_A)}{(2N - s - \Delta s - a + 1)(2N - a - s + 1)}. \tag{77}$$

Also the net return spread between the index and active funds can be written as

$$\delta_I(\omega) + \bar{\phi}_I - \bar{\phi}_A(\omega) - \delta_A \tag{78}$$

$$= \delta_I(\omega) + \bar{\phi}_I - \delta_A - \frac{N(\delta_I(\omega) + \bar{\phi}_I - \delta_A)}{2N - a - s + 1}, \tag{79}$$

$$= \frac{(N - a - s + 1)(\delta_I(\omega) + \bar{\phi}_I - \delta_A)}{2N - a - s + 1}. \tag{80}$$

To see that AUM per fund increases with more selectors, it is sufficient to show that

$$\frac{\delta_I(\omega') + \bar{\phi}_I - \bar{\phi}_A(\omega') - \delta_A}{N - s - \Delta s - a} > \frac{\delta_I(\omega) + \bar{\phi}_I - \bar{\phi}_A(\omega) - \delta_A}{N - a - s}, \tag{81}$$

$$(N - a - s)(\delta_I(\omega') + \bar{\phi}_I - \bar{\phi}_A(\omega') - \delta_A) > (N - s - \Delta s - a)(\delta_I(\omega) + \bar{\phi}_I - \bar{\phi}_A(\omega) - \delta_A), \tag{82}$$

$$(N - a - s)(\bar{\phi}_A(\omega') - \bar{\phi}_A(\omega)) < \Delta s(\delta_I(\omega) + \bar{\phi}_I - \bar{\phi}_A(\omega) - \delta_A), \tag{83}$$

$$\frac{\Delta s(N - a - s)N(\delta_I(\omega) + \bar{\phi}_I - \delta_A)}{(2N - s - \Delta s - a + 1)(2N - a - s + 1)} < \frac{\Delta s(N - a - s + 1)(\delta_I(\omega) + \bar{\phi}_I - \delta_A)}{2N - a - s + 1}, \tag{84}$$

$$\frac{(N - a - s)N}{(N - a - s + 1)(2N - s - \Delta s - a + 1)} < 1, \tag{85}$$

where the last line follows because the total of selectors and activists is always less than $N$, the number of industries.

□

*Proof of Proposition 7.* Beginning with the expressions for AUM per fund before (right) and after (left) the transition, per Equation (76), we assume the result and show that it holds:

$$\frac{N(\delta_I(\omega') + \bar{\phi}_I - \bar{\phi}_A(\omega') - \delta_A)}{\gamma\sigma(N - s - \Delta s - n + \Delta s)} > \frac{N(\delta_I(\omega) + \bar{\phi}_I - \bar{\phi}_A(\omega) - \delta_A)}{\gamma\sigma(N - a - s)}, \tag{86}$$

$$\delta_I(\omega') - \bar{\phi}_A(\omega') > \delta_I(\omega) - \bar{\phi}_A(\omega), \tag{87}$$

$$\delta_I(\omega') - \delta_I(\omega) > \bar{\phi}_A(\omega') - \bar{\phi}_A(\omega), \tag{88}$$

$$(2N - a - s + 1)(\delta_I(\omega') - \delta_I(\omega)) > N(\delta_I(\omega') - (\delta_I(\omega)), \tag{89}$$

$$\frac{2N - a - s + 1}{N} > 1. \tag{90}$$

Since the number of funds remains the same and AUM per fund increases, total AUM invested with actively managed funds increases. □

*Proof of Proposition 8.* This simply requires examination of the partial derivatives of optimal fees with respect to $\bar{\phi}_I$:

$$\frac{\partial \bar{\phi}_A}{\partial \bar{\phi}_I} = \frac{\partial \bar{\phi}_S}{\partial \bar{\phi}_I} = \frac{(2(N-a-s)+1)N}{(2N-s-a+1)(2(N-a-s)+1)} > 0. \qquad \square$$

*Proof of Proposition 9.* From Equations (25) and (26),

$$\bar{\phi}_A(\omega) - \bar{\phi}_S(\omega) = \frac{(N-a-s)(\delta_S - \delta_A)}{2(N-a-s)+1}.$$

Since $2(N-a-s)+1 > 0$ always and $N-a-s > 0$ by assumption, the sign of the difference in fees is given by $\delta_S - \delta_A$. $\square$

*Proof of Proposition 10.* Since $\delta_S > \delta_A$ and $a+s < N$, the result follows immediately because

$$\bar{\phi}_A(\omega) - \bar{\phi}_S(\omega) = \frac{N-a-s}{2(N-a-s)+1}(\delta_S - \delta_A) < \delta_S - \delta_A. \qquad \square$$

*Proof of Proposition 11.* The difference in fees is

$$\bar{\phi}_A(\omega') - \bar{\phi}_A(\omega) = \bar{\phi}_S(\omega') - \bar{\phi}_S(\omega) \tag{91}$$

$$= \frac{(2(N-a-s)+1)\Delta s(\delta - \delta_A) - (N-a-s+1)\Delta s(\delta_S - \delta_A)}{(2N-s-a+1)(2(N-a-s)+1)} \tag{92}$$

$$> \frac{(2(N-a-s)+1)\Delta s(\delta - \delta_A) - (N-a-s+1)\Delta s(\delta - \delta_A)}{(2N-s-a+1)(2(N-a-s)+1)} \tag{93}$$

$$= \frac{(N-a-s)\Delta s(\delta - \delta_A)}{(2N-s-a+1)(2(N-a-s)+1)} \geq 0. \tag{94}$$

The inequality in the third line follows from the standing assumption $\delta_S < \delta$. The change in the index depreciation rate is

$$\delta_I(\omega') - \delta_I(\omega) = \frac{\Delta s(\delta - \delta_A)}{N} > 0. \tag{95}$$

$\square$