

Disagreement and Control Rights: Implications for Debt Policy and Aggregate Dynamics*

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Abstract

We examine firm capital structure when heterogeneous agents optimally hold different claims, and control of the firm may change hands. When agents cannot commit to firm value maximization, controlling agents have the incentive to alter firm policy to maximize their preferred portfolio at the expense of other claim-holders. We consider settings that can include partial control rights to minority share-holders and/or debt-holders. In general equilibrium, the distortions relative to complete contracting are large even with small disagreement. However, it need not be the case that the distortions amplify the business cycle nor that stronger protection of debt holders mitigates the problem.

1 Introduction

What are the implications for firm policies of having diverse investors? This is a broad and important question for understanding corporate behavior and the implications of alternative legal and regulatory frameworks. While the problem admits many potential formulations, central to any of them is the observation that control of the firm will, in general, change hands over time. Under the realistic assumption that firms cannot *ex ante* restrict the actions of all future potential owners, we should then expect that, in general, (1) firm policies may change suddenly as the composition of its investors evolves, and (2) the prices of firms' claims

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will reflect the benefits and risks of such changes in policy. Thus, the dynamics of control rights may have first-order implications at both the firm and aggregate levels.

We study the properties of one firm policy – capital structure determination – in a setting where control of the firm (and determination of the policy) can pass between agents of two types who disagree about the optimal degree of leverage. In this case, the disagreement directly affects the supply of debt and equity, as well as their prices. In general equilibrium, the quantity of debt affects aggregate risk which feeds back into firm policies via state prices. Our model enables us to speak to important questions about the potential distortionary effects of debt, combined with incomplete contracting, on the macroeconomy.

In our framework, agents have homogeneous preferences but differing beliefs about the likelihood of an economic downturn. The state of the economy is summarized by the relative wealth of the agents. Financial markets are effectively complete with respect to aggregate risk, but firms cannot write contracts covering their individual owner/managers' future actions. The capital structure problem is a standard trade-off framework. Firms are stochastically identical, enabling us to close the model. We fully characterize the dynamics of the wealth distribution, of leverage, and of aggregate default risk, as well as the portfolios of the agents, and the prices and risk premia of the firm's claims.

The benchmark for understanding our economy is the special case where all agents can commit *ex ante* to all future policies. In this case, debt issuance is fairly insensitive to the relative wealth of optimistic versus pessimistic investors. This is because all agents agree on total value maximization — i.e., of debt and equity combined — as the objective of initial debt issuance, assuming complete financial markets. Furthermore they agree on the amount of debt that achieves this objective. Therefore, in this setting, the allocation of control rights is irrelevant. This result obtains despite disagreement about the likelihood of bad outcomes and attendant financial distress.

If firms cannot commit to a given capital structure, however, leverage becomes very sensitive to the relative wealth of optimistic versus pessimistic investors. Deviations from

efficient, value-maximizing debt can be large, and a small change in relative wealth may alter equilibrium debt from inefficiently low to inefficiently high. In this setting, the allocation of control rights is pivotal.

The starkest example is the case in which the majority equity shareholder completely controls the firm. In this case, there is an interior threshold value of the relative wealth state-variable at which *ex post* control shifts from the one agent to the other. As a consequence, equilibrium capital structure may be radically different on one side of the threshold versus the other. Optimists and pessimists choose different portfolios of debt and equity, and will use control of the firm to maximize their personal portfolio values, not total firm value. Wealthy pessimists may hold a majority equity stake despite allocating most of their wealth to debt, and therefore they would choose to retire some debt *ex post*. By contrast optimists allocate relatively more of their wealth to equity, and have an *ex post* motive to issue more debt. In either case, gains come at the expense of the minority shareholder. Prices and interest rates then also change abruptly at the control threshold.

We augment our framework to model arguably more realistic settings where minority shareholders may affect the firm's objectives (e.g., via protected board seats) and/or bondholders may have some degree of protection (e.g., via covenants), which also ensures continuity of the model solution around the threshold where majority ownership changes hands. While we model these protections in reduced-form, we are able to solve fully for all quantities and prices. Agents anticipate the shift in control and firm objectives as the wealth distribution evolves. The results in these cases qualitatively resemble the example with majority equity control, but with less abrupt changes. Firm leverage shifts from countercyclical to procyclical and then to countercyclical again, as in an S-shaped curve. With sufficient minority shareholder protection, the distortion relative to value-maximization becomes small, but interestingly this is not the case for debtholder protection. Instead, increasing debtholder protection can lead to increasingly inefficient under-leveraging. This conclusion contradicts the belief that countering equity holders' incentives to over-lever ought to strictly improve

aggregate outcomes.

Interpreting positive aggregate shocks as “good times” or expansions, a notable and counterintuitive finding from the case with weak minority protections is that when optimists gain control and increase leverage, they do this immediately, since that is when the minority pessimist’s wealth is largest and thus presents the greatest expropriation incentive. If good times continue, this incentive fades, and the controlling shareholder will optimally lower leverage. Thus the model features regions of the state space in which leverage can be countercyclical while in others it is procyclical. Strengthening minority protections makes the initial leveraging following booms more gradual, and limits its extent, as we show via comparative statics.

Two main ingredients of our model, heterogeneous investors and limited commitment, act in tandem to generate meaningful variation in aggregate leverage in general equilibrium. Absent investor heterogeneity, limited commitment has no bite, as all investors optimally hold identical portfolios which must be proportional to total firm value in equilibrium. No *ex post* incentive to diverge from value maximization emerges. Without limited commitment, we show that heterogeneous beliefs have a large impact on asset prices and the debt-to-equity ratio through discount rates, but this translates into small variation in capital structure as captured by the interest-to-output ratio, default probability, and credit spreads, with little impact on the severity of economic downturns. When limited commitment and investor heterogeneity are combined, divergence from value-maximizing leverage can be volatile and economically significant along all of these dimensions.

1.1 Related literature

An extensive literature documents the importance of heterogeneous beliefs for aggregate and cross-sectional asset price dynamics. Empirically, Diether et al. (2002) use analyst forecasts to proxy for difference of opinion about stocks, and show that belief dispersion negatively predicts returns in the cross section. Anderson et al. (2005) connect such findings to the-

oretical underpinnings, incorporating analyst forecast dispersion in structural estimation. Buraschi et al. (2014) extend the study of firm-level disagreement to the joint determination of stock returns and credit spreads. Relevant to our study, they control for firm-level leverage, and show theoretically and empirically that the directional effects of disagreement shocks depend on the degree of leverage.

Gao et al. (2019) study time-varying aggregate disagreement as a macroeconomic risk factor, using a measure based on the Blue Chip Economic Indicators. They show that exposure to disagreement — disagreement beta — carries a risk premium in US stocks, corporate bonds, mortgage backed securities, and US government securities. Giacoletti et al. (2021) and Singleton (2021) study disagreement about output, inflation, and future yields as forecast variables for US treasury returns, using both Blue Chip and Survey of Professional Forecasters data. Chernov and Bauer (forthcoming) forecast US Treasury returns using option-implied skewness, and argue that its predictive power originates in heterogeneous beliefs proxied by disparate professional forecasts.

Theoretical work also argues that disagreement is an important contributor to risk premium and macroeconomic dynamics in general equilibrium. Detemple and Murthy (1994) conduct foundational work on disagreement in a frictionless production economy, whereas Basak (2005) studies an endowment economy, and Dieckmann (2011) combines disagreement with rare events, or jumps. Dumas et al. (2009) introduce a theoretical framework for time-varying disagreement, or sentiment, as a driver of broad market volatility. Ehling et al. (2018) study time-varying variation in disagreement about inflation, providing theoretical and empirical evidence that disagreement impacts nominal yields through its effect on the real economy. Baker et al. (2016) highlights the potential for disagreement to amplify or prolong recessions through its effect on aggregate investment.

Despite this broad and deep literature, there is little empirical work and perhaps no theoretical work suggesting control rights as a mechanism by which disagreement affects asset prices, aggregate debt, and related disaster risk. In theoretical work surveyed above,

disagreement essentially acts through state prices, which may in turn affect cross-sectional consumption volatility, investment, and growth. Our model includes the state price channel, but agents also exercise control rights to alter the corporate objective. The control rights mechanism has a potent impact on aggregate corporate debt dynamics.

Although the control rights channel has received little attention in the theoretical asset pricing literature, it is central to much work in corporate finance, including several studies incorporating disagreement. Our work differs from this line of research in two key respects. First, we study disagreement among investors, as opposed to disagreement between firm managers and investors. Second, our model is closed and all agents are risk-averse.

Jung and Subramanian (2014), for example, model the effect of managerial optimism on long-and-short-term debt issuance. They find a shift from long term debt to short term debt as managerial optimism increases, reflecting a tradeoff between incentive and risk-sharing effects. Blockholders are distinguished from small shareholders, but all investors have identical beliefs. Boot and Thakor (2011) model initial owners and managers, who agree, and new investors with different priors about project quality. They find a pecking-order for funding with cash, equity, and debt, which reflects the manager's preference for ex-post autonomy. Dittmar and Thakor (2007) study a model with debt and equity only, and emphasize different beliefs of managers and investors as a rationalization for equity issuance when stock prices are high. All of these studies are set in partial equilibrium, and investors are risk-neutral, as the emphasis is on managerial incentives and preferences rather than equilibrium price determination.

Another body of work models deviations from firm value maximization in dynamic general equilibrium, but abstracts from heterogeneous investors and control rights conveyed by ownership of securities. For example, Johnson (2021) and Johnson et al. (2020) study firm leverage dynamics in a DSGE framework in which the representative household maximizes equity value, rather than total firm value, for exogenous reasons. Endogenizing the firm's objective is a key contribution of our study, and accounting for owners' common exposure to

debt and equity leads to more nuanced results. Empirically, Ivashina and Sun (2011) provide evidence of common ownership of debt and equity by institution investors, and Chu et al. (2020) show that common ownership of debt and equity helps alleviate conflicts between debt and equity holders.

Relative to the extant work above, our contribution is to present, to our knowledge, the first capital structure dynamics when control rights vary endogenously as a result of trade between heterogeneous investors in general equilibrium. We adopt a simple but flexible control rights framework in which debt holders as well as minority shareholders can weigh in corporate decisions. Changes in the firm's objective interact with changes in state-prices, with novel implications for firm behavior and macroeconomics dynamics.

The outline of the paper is as follows. The next section described the model and discusses the construction of equilibrium solutions. Section 3 analyzes the main properties of optimal investor portfolios and the dynamics of control and leverage. Section 4 considers the model's implications for aggregate dynamics, and discusses welfare and policy implications. A final section summarizes and concludes.

2 Model

The model is set in continuous time. Two infinitely lived competitive agents, a and b , have identical log utility, but dogmatically disagree about the arrival rate of jumps, which destroy an endogenous fraction of aggregate consumption. They trade portfolios of stock and bonds in a continuum of ex-ante identical firms, which pay state-contingent coupons and dividends. Due to their different perceptions of jump risk, the agents will generally hold different portfolios, which also vary over time with the economic state.

Stocks (and in some scenarios bonds) also carry control rights, which determine firm management objectives and hence affect equilibrium capital structure. Although we abstract from principal-agent frictions and assume that firm managers act in the best interests of

their controlling shareholders, shareholder interests are divergent. All agents would favor total firm value maximization ex-ante. Ex-post, a controlling shareholder with, e.g., more exposure to stock than bonds might prefer to increase his total portfolio value by issuing additional bonds, devaluing existing bonds. Because firm managers cannot commit to a given capital structure policy and will acquiesce to controlling shareholder demands, equilibrium capital structure generally diverges from value-maximization, because the non-controlling agent anticipates the expropriation incentive of the controlling agent.

Under some assumptions regarding how firms choose capital structure (to be discussed later), all firms adopt the same capital structure policy, and it is optimal for investors to trade fully diversified portfolios of stocks and corporate bonds. Only aggregate risk is priced and this risk is spanned by the available assets. In this respect markets are complete, which in combination with our assumption of competitive price-taking agents allows us to recast the aggregated problem as a social planners problem, taking firm leverage as given.

We begin by solving this comparatively simple planner's problem, which yields an optimal consumption sharing rule and state price density. Then we decentralize the solution by solving for portfolios of stock and bonds that implement each agent's optimal consumption rule. Portfolio holdings determine the control rights assigned to each agent, which determine firm managers' objectives in choosing leverage, thus closing the model.

2.1 Preferences, consumption, and state prices

The flow rate of aggregate consumption, which the planner takes as given, is

$$C_t = \delta(\gamma_t)Y_t \text{ where } \delta(\gamma_t) = (1 - \tau) + \tau\gamma_t. \quad (1)$$

Y_t is aggregate output, of which a fraction $\delta(\gamma_t) \in (0, 1)$ remains after dissipative taxation at rate τ . Because debt provides a tax shield, $\delta(\gamma_t)$ is increasing in aggregate leverage γ_t .

Aggregate output Y_t follows

$$\frac{dY_t}{Y_t} = \mu dt + (\zeta(\gamma_t) - 1)dJ_t^j, \quad (2)$$

where μ is a growth parameter, $\zeta(\gamma_t) \in (0, 1]$ is aggregate output remaining after a jump in J_t^j , and $j \in a, b$ indicates the agent under whose measure we write the jump process. Both agents agree on $\zeta(\gamma_t)$, which is decreasing in γ_t . The difference between the two agents is that they perceive different Poisson arrival rates λ_j for the jump.

Without loss of generality we assume $\lambda_a < \lambda_b$: agent a is relatively optimistic and b is relatively pessimistic. We will write the model under agent b 's beliefs and omit the measure superscript unless necessary for clarity, e.g., J_t for J_t^b .¹

The planner's problem is

$$\sup_{C_{a,t}, C_{b,t}} \mathbb{E}_0 \left[\int_0^\infty \eta_t e^{-\beta t} \log(C_{a,t}) + e^{-\beta t} \log(C_{b,t}) dt \right], \quad (3)$$

subject to

$$C_{a,t} + C_{b,t} = C_t, \quad (4)$$

$$\frac{d\eta_t}{\eta_t} = (\lambda_b - \lambda_a)dt + \left(\frac{\lambda_a}{\lambda_b} - 1 \right) dJ_t. \quad (5)$$

The process η_t implements the change of measure from agent b 's beliefs to agent a 's.

Define the state-contingent Pareto share of agent a

$$x_t \equiv \frac{\eta_t}{\eta_t + 1}. \quad (6)$$

Agent a 's initial Pareto share $x_0 \in [0, 1]$ is exogenous, but variation in x_t is driven by the difference of beliefs.² Intuitively, x_t increases in the absence of jumps (which a considers

¹This choice is also without loss of generality, and does not imply that agent b has correct beliefs.

²Since our model is time invariant, the initial Pareto share is not important to how we solve for equilibrium,

relatively unlikely), but decreases if and when a jump occurs, remaining in the unit interval.

It follows from the planner's first order condition that the optimal consumption sharing rule is

$$C_{a,t} = x_t C_t, \quad (7)$$

$$C_{b,t} = (1 - x_t) C_t. \quad (8)$$

Under log utility as in our setting, an agent's Pareto share is also his consumption share.

The state price density equals the marginal utility of consumption. Per the planner's FOC, marginal utility for agents a and b is equalized up to a change of measure. Under our choice of b 's beliefs for the default measure, the pricing kernel is

$$\xi_t = \frac{e^{-\beta t}}{C_{b,t}} = \frac{e^{-\beta t}}{(1 - x_t)\delta(\gamma_t)Y_t}, \quad (9)$$

with dynamics

$$\frac{d\xi_t}{\xi_t} = -r_t dt - \lambda_b(\zeta_{\xi,t} - 1)dt + (\zeta_{\xi,t} - 1)dJ_t. \quad (10)$$

We defer statement of the equilibrium interest rate r_t , SPD jump coefficient $\zeta_{\xi,t}$, and equivalent coefficients for price processes until after the model is fully described.

2.2 Wealth, securities returns, and portfolios

Denote each agent's wealth $X_{j,t}$, for $j \in \{a, b\}$. Agent b 's wealth is like a security that pays a dividend equal to $C_{b,t}$, with price

$$\begin{aligned} X_{b,t} &= \mathbb{E}_t \left[\int_t^\infty \frac{\xi_s^b}{\xi_t^b} C_{b,s} ds \right] \\ &= \frac{C_{b,t}}{\beta}. \end{aligned} \quad (11)$$

except when $x_0 = 0$ or $x_0 = 1$, such that the model collapses to a single agent setting without disagreement. Under the conventional assumption that the change of measure has initial value $\eta_0 = 1$, then $x_0 = 1/2$.

Similarly a 's wealth is

$$X_{a,t} = \frac{C_{a,t}}{\beta}, \quad (12)$$

and aggregate wealth, equal to the value of the firm, is

$$X_t = P_t = \frac{C_t}{\beta}. \quad (13)$$

It follows that consumption and wealth dynamics are identical.

For drift coefficient $\mu_{X_{b,t}}$ and jump coefficient $\zeta_{X_{b,t}}$ to be determined, agent b 's wealth has dynamics

$$\frac{dX_{b,t}}{X_{b,t}} = \mu_{X_{b,t}}dt + (\zeta_{X_{b,t}} - 1)dJ_t. \quad (14)$$

Although agents may trade stock and bonds in any of a continuum of firms, it is sufficient to consider each agent's choice of diversified portfolios of stock and bonds, or equivalently the stock and bonds of a representative firm. We defer the details of how individual firms choose capital structure and how securities aggregate until later.

Specifically the representative firm issues a floating rate note with interest rate \hat{r}_t . A unit investment in the bond maintains its value until a jump occurs, which causes a fraction of the underlying firms to default, leading to aggregate fractional loss $1 - \zeta_{B,t}$. The representative bond return process is therefore

$$\hat{r}_t dt + (\zeta_{B,t} - 1)dJ_t, \quad (15)$$

where the corporate floating rate is

$$\hat{r}_t = r_t + (\zeta_{\xi,t} - 1)\lambda_b - (\zeta_{B,t}\zeta_{\xi,t} - 1)\lambda_b. \quad (16)$$

The remaining output flow, net of taxes and interest, is paid to stockholders. The

aggregate return process for stocks is

$$(\mu_{S,t} + \hat{\delta}_t)dt + (\zeta_{S,t} - 1)dJ_t, \quad (17)$$

where $1 - \zeta_{S,t}$ is the aggregate fractional loss to stockholders following a jump, $\mu_{S,t}$ is the stock price drift coefficient, $\hat{\delta}_t$ is the dividend yield, and price appreciation plus dividend yield satisfy

$$\mu_{S,t} + \hat{\delta}_t = r_t + (\zeta_{\xi,t} - 1)\lambda_b - (\zeta_{S,t}\zeta_{\xi,t} - 1)\lambda_b. \quad (18)$$

Agents a and b choose exposure to aggregate corporate stock and debt to implement their respective optimal wealth processes. To calculate portfolios for agents a and b , we replicate their exposure to jump risk using the stock and the bond.³ For each agent $j \in \{a, b\}$, let $\phi_{B,t}^j$ be his fraction of wealth invested in bonds, and $\phi_{S,t}^j = 1 - \phi_{B,t}^j$ his fraction of wealth invested in stocks. Then each agent's portfolio satisfies

$$\begin{aligned} \zeta_{X_j,t} &= \phi_{B,t}^j \zeta_{B,t} + (1 - \phi_{B,t}^j) \zeta_{S,t}, \\ \Rightarrow \phi_{B,t}^j &= \frac{\zeta_{X_j,t} - \zeta_{S,t}}{\zeta_{B,t} - \zeta_{S,t}}. \end{aligned} \quad (19)$$

2.3 Firms and capital structure

We now turn to the capital structure decision of an individual firm manager. There is a unit mass of ex-ante identical firms, indexed i . Before tax output of firm i follows

$$\frac{dY_{i,t}}{Y_{i,t}} = \mu dt + (\tilde{\zeta}_i(\gamma_{i,t}) - 1)dJ_t. \quad (20)$$

J_t is a common jump process: all firms experience jumps at the same time, so they are the source of aggregate risk. However the recovery rate of output $\tilde{\zeta}_i \in [0, 1]$ is firm-specific. The variable $\gamma_{i,t}$, which is the firm's coupon as a fraction of output, controls the firm's capital

³A condition of equilibrium is that these portfolios also replicate the drift of each agent's optimal wealth process.

structure.

Corporate profits, equivalent to dividends in our model, are taxed at rate τ . After tax output is

$$\delta(\gamma_{i,t})Y_{i,t}, \text{ where } \delta(\gamma_{i,t}) = (1 - \tau) + \tau\gamma_{i,t}. \quad (21)$$

In choosing capital structure, firm managers face the standard trade-off between a tax shield and default risk, each of which increases with the firm's leverage. A firm enters technical default if it cannot pay its coupon immediately following a jump. We assume that corporate debt promises coupon $\gamma_{i,t}Y_{i,t}$, which is continuously adjustable without cost in normal times. However in the event of a jump the coupon cannot instantaneously (and discontinuously) adjust downward to prevent a technical default.

Let \tilde{L}_i be the fraction of output recovered by firm i following a jump before any costs of distress. For each jump and for each firm i , \tilde{L}_i is drawn independently from a power distribution with common parameter α .⁴ The density, cumulative distribution, and moments of \tilde{L}_i are

$$f(L) = \alpha L^{\alpha-1}, F(L) = L^\alpha, \text{ and } \mathbb{E}[L^n] = \frac{\alpha}{\alpha + n}, \quad (22)$$

respectively.

If a jump occurs at time t and the realization L_i is less than the coupon rate immediately prior to the jump $\gamma_{i,t-}$, then firm i defaults and incurs additional fractional loss of output $1 - Z$ representing costs of financial distress. $Z \in (0, 1)$ is a constant common to all firms. Therefore the realized firm-specific recovery rate is

$$\zeta_i(\gamma_{i,t-}) = \begin{cases} ZL_i & \text{if } L_i < \gamma_{i,t-}, \\ L_i & \text{otherwise.} \end{cases} \quad (23)$$

Conditional on $\gamma_{i,t-}$, the default probability π and the expected unconditional recovery

⁴The power distribution allows for losses $1 - L_i$ with fat tails, and has been used to model rare disasters, e.g., in Pindyck and Wang (2013) and Baker et al. (2020).

rate ζ are

$$\pi = \gamma_{i,t-}^\alpha, \quad (24)$$

$$\zeta = \frac{\alpha}{\alpha + 1} (1 - (1 - Z)\gamma_{i,t-}^{\alpha+1}). \quad (25)$$

At each date and state the manager of firm i acts according to the interests of the current controlling owners, but he cannot commit to maintain a particular capital structure policy into the future. We will solve a general problem in which the manager considers both majority and minority owner interests, and ownership (i.e., portfolio holdings) change over time. The equilibrium state-contingent capital structure is a fixed-point such that the manager has no incentive to deviate.

However to provide some intuition with minimal notational encumbrance, we will initially drop the time and firm subscripts t and i , respectively and assume that one agent has complete control of the firm. Let $\hat{\phi}_S^j$ be the ex-ante fraction of stock held by the controlling owner j , with $\hat{\phi}_B^j$ the ex-ante fraction of bonds.⁵ Agent j wishes to adjust the current capital structure to maximize the value of his portfolio.

Suppose agent j issues new debt B^{new} , and uses the proceeds to pay a dividend to stockholders. Writing P for the total firm value and B for the value of the firm's old (ex-ante) debt, the value of the stock inclusive the dividend is

$$P - (B^{new} + B) + B^{new} = P - B.$$

We can interpret a negative value of B^{new} as raising capital from stockholders to retire debt without altering the result on the right hand side. Therefore a change in capital structure affects the value of the firm's stock by changing total firm value or the value of old debt, but the value of new debt does not enter directly.

⁵Note that $\hat{\phi}_S^j$ and $\hat{\phi}_B^j$ are in different units from ϕ_S^j and ϕ_B^j introduced earlier, which represented the fraction of agent j 's *wealth* invested in aggregate stock and bonds.

The value of the controlling owner's portfolio after capital structure changes is

$$\hat{\phi}_S^j(P - B) + \hat{\phi}_B^j B = \hat{\phi}_S^j(P + \rho^j B), \text{ where } \rho^j = \frac{\hat{\phi}_B^j - \hat{\phi}_S^j}{\hat{\phi}_S^j}.$$

Variable ρ^j can be interpreted as agent j 's expropriation incentive. Holding firm value constant, if $\rho < 0$, then agent j has an incentive to decrease the value of old debt (increasing the value of his stock at the expense of creditors), whereas if $\rho > 0$ then agent j has an incentive to increase the value of old debt (increasing the value of his bonds at the expense of stockholders).

Now consider a generalization of the above problem in which minority and majority controlling interests are given weight:

$$\begin{aligned} w\hat{\phi}_S^a(P + \rho^a B) + (1 - w)\hat{\phi}_S^b(P + \rho^b B) \\ \equiv (w\hat{\phi}_S^a + (1 - w)\hat{\phi}_S^b)(P + \rho B), \end{aligned} \tag{26}$$

where $w \in [0, 1]$ and

$$\rho = \frac{w(\hat{\phi}_B^a - \hat{\phi}_S^a) + (1 - w)(\hat{\phi}_B^b - \hat{\phi}_S^b)}{w\hat{\phi}_S^a + (1 - w)\hat{\phi}_S^b}. \tag{27}$$

The above is similar to the objective of a single controlling owner, but with ρ redefined as a weighted average of each agent's interests.⁶

Weights are determined by portfolio holdings in combination with two parameters. First, a fraction $\psi \in [0, 1]$ of control rights is assigned to stockholders, with the remaining fraction $1 - \psi$ assigned to bondholders. Although we have in mind $\psi > 1/2$, such that relatively more control is assigned to stockholders, bondholders also exert control, e.g., via the use of covenants, which we capture in reduced form by allowing $1 - \psi > 0$. Incorporating parameter ψ and bounding stock and bond holdings between zero and one, we calculate each agent's

⁶Our definition of w , in equation Equation (29), does not rule out zero or negative values in the denominator of ρ in Equation (27). The denominator is positive in subsequent numerical examples.

fractional control rights

$$w_j = \psi(\max(\min(\hat{\phi}_S^j, 1), 0)) + (1 - \psi)(\max(\min(\hat{\phi}_B^j, 1), 0)), \text{ for } j \in \{a, b\}. \quad (28)$$

Second, parameter $\theta \in [0, \infty)$ adjusts the relative power of the controlling majority. Agent a 's adjusted control weight is

$$w = \frac{w_a^\theta}{w_a^\theta + w_b^\theta}, \quad (29)$$

and agent b 's weight is $1 - w$. Letting $\theta \rightarrow \infty$ models strict majority control, since $w \rightarrow 1$ if $w_a > w_b$ or $w \rightarrow 0$ if $w_a < w_b$. Letting $\theta = 0$ models an egalitarian objective with $w = 1/2$ regardless of agent portfolio holdings.⁷

Restoring time and firm subscripts, at the start of each period firm i 's manager solves

$$\max_{\gamma_{i,t}} P_{i,t} + \rho_t B_{i,t}. \quad (30)$$

In solving the above problem, the firm manager takes as given ex-ante portfolio holdings (i.e., ρ_t) and the scale of old debt, but he can alter the present value of the firm and the present value of old debt by adjusting future capital structure. When $\theta = 0$, $\rho = 0$: an egalitarian manager maximizes total firm value, nesting value-maximization within our limited commitment framework.

The next section details and solves the managers' problem as part of the general equilibrium.

3 Securities Markets and Equilibrium

We restrict attention to symmetrical equilibria in which each firm i optimally chooses the same capital structure $\gamma_{i,t} = \gamma_t$. Infinitely lived households imply a time-invariant model

⁷We adopt the convention $0^0 = 1$ in the event that w_a or w_b is identically zero.

solution, and since preferences are homothetic we anticipate securities prices that are homogeneous in aggregate output Y_t (or firm output $Y_{i,t}$). We therefore drop time subscripts, and posit subject to verification that price ratios and capital structure can be written as functions of the Pareto share x , e.g., capital structure is described by $\gamma(x)$ such that $\gamma_t = \gamma(x_t)$, $\forall t$.

We first establish some results for prices that hold for any common policy $\gamma_i(x) = \gamma(x)$, $\forall i$. An immediate implication of common capital structure is that the conditional aggregate default rate $\pi(x)$ and aggregate recovery rate $\zeta(x)$ are equal to the expected default rate and expected recovery rate for individual firms, as given in Equations (24) through (25).

Equation (13) shows that the value of the representative firm (aggregate wealth) can be written as

$$P(x, Y) = p(x)Y, \text{ where the price-output ratio is} \tag{31}$$

$$p(x) = \frac{\delta(x)}{\beta}.$$

The value of any firm i with current output Y_i is given by $P_i = P(x, Y_i)$. Since idiosyncratic risk is not priced and each firm has identical exposure to aggregate jump risk, all firms have identical output-capital ratios matching that of the representative firm.⁸

Corporate debt has the following characteristics:

1. All firms issue floating rate notes promising coupon rate $\hat{r}(x)$ in state x .
2. All debt is of equal seniority.
3. Debt is subject only to technical default, as described in Section 2.3.
4. $\hat{r}(x)$ is defined such that a unit investment in debt of a firm with capital structure policy $\gamma(x)$ maintains its value outside of default.

Our standardized debt contract compensates investors for changes in aggregate default risk, since $\hat{r}(x_t)$ adjusts with $\gamma(x)$. However if the manager of firm i deviated from the

⁸See Appendix A for details.

representative firm's policy by choosing $\gamma_i(x) \neq \gamma(x)$, the coupon on firm i 's debt would not adjust, and the value of firm i 's debt would deviate from par. This is the origin of distortions due to limited commitment, since firm managers can commit to pay a broad market interest rate, but cannot make contracts contingent on idiosyncratic deviations from promised capital structure.

Because of limited commitment, the distribution of control rights alters equilibrium capital structure, since $\gamma(x)$ adjusts until no firm has an incentive to deviate from aggregate policy in any state x . Since deviation from aggregate policy is an off-equilibrium outcome, debt in individual firms shares a common ratio to output with debt in the representative firm in equilibrium.

In state x , the representative firm with output Y adjusts its debt such that the total coupon flow is $\gamma(x)Y$. Since corporate debt pays floating interest rate $\hat{r}(x)$, it follows that the value aggregate debt is

$$B(x, Y) = b(x)Y, \text{ where the debt-output ratio is} \tag{32}$$

$$b(x) = \frac{\gamma(x)}{\hat{r}(x)}.$$

The debt-output ratio follows because by definition one numeraire unit of floating rate debt pays coupon $\hat{r}(x)$ in state x .

Similarly, debt issued by firm i with output Y_i has value $B(x, Y_i)$ in state x .

Finally, stock is worth firm value less the value of debt, or

$$S(x, Y) = (p(x) - b(x))Y. \tag{33}$$

It remains to solve for interest rates and equilibrium returns as functions of the state

variable x given policy function $\gamma(x)$. The Pareto weight x follows

$$\begin{aligned} \frac{dx_t}{x_t} &= (1-x_t)(\lambda_b - \lambda_a)dt + (\zeta_x(x_t) - 1) dJ_t^b, \text{ where} \\ \zeta_x(x_t) &= \frac{\lambda_a}{\lambda_a x_t + \lambda_b(1-x_t)}. \end{aligned} \quad (34)$$

The coefficient $\zeta_x(x)$ summarizes the state transition following a jump: if agent a has Pareto weight x immediately before a jump occurs, then his Pareto weight is $\zeta_x(x)x$ immediately after the jump.

Suppressing the time argument, the SPD can be written

$$\xi(x, Y) = \frac{e^{-\beta t}}{(1-x)\delta(x)Y}. \quad (35)$$

The risk-free rate is

$$r(x) = \beta + \mu + \left((1-x) \frac{\delta'(x)}{\delta(x)} - 1 \right) x(\lambda_b - \lambda_a) - \lambda_b(\zeta_\xi(x) - 1), \quad (36)$$

in which the jump coefficient is

$$\zeta_\xi(x) = \frac{\xi(\zeta_x(x)x, \zeta(x)Y)}{\xi(x, Y)} = \frac{(1-x)\delta(x)}{(1-\zeta_x(x)x)\delta(\zeta_x(x)x)\zeta(x)}, \quad (37)$$

and the derivative of after-tax output with respect to x is

$$\delta'(x) = \tau\gamma'(x). \quad (38)$$

The corporate floating rate $\hat{r}(x)$ is recursively dependent on the exposure of bonds to default losses. When a jump occurs, an individual firm defaults with probability $\pi(x)$, otherwise its bonds remain valued at par. Due to our assumption of equal seniority, all creditors of a given firm suffer equal percentage losses in the event of default. Therefore the percentage loss in default is equal to the percentage decline in the total value of the firm's

bonds from the moment before the jump until the moment after the jump. Prior to the jump, the firm's debt is worth $B(x, Y_i)$, whereas afterwards and conditional on default the value of debt is equal to residual firm value $P(\zeta_x(x)x, Z\tilde{\zeta}_i(x)Y_i)$. Therefore the conditional expected bond recovery rate reduces to

$$\zeta_B(x) = 1 - \pi(x) + \pi(x) \frac{\zeta_D(x)\delta(\zeta_x(x)x)\hat{r}(x)}{\beta\gamma(x)}, \quad (39)$$

for ζ_D defined in Appendix A.

Since idiosyncratic risk is not priced in equilibrium, ζ_B , which is also the certain conditional recovery rate for the representative firm's bonds, is the relevant risk measure for the equilibrium corporate floating rate. The corporate floating rate is

$$\begin{aligned} \hat{r}(x) &= r(x) + (\zeta_\xi(x) - 1)\lambda_b - (\zeta_B(x)\zeta_\xi(x) - 1)\lambda_b, \\ \hat{r}(x) &= \left[\beta + \mu + \left((1-x) \frac{\delta'(x)}{\delta(x)} - 1 \right) x(\lambda_b - \lambda_a) \right. \\ &\quad \left. - ((1 - \pi(x))\zeta_\xi(x) - 1)\lambda_b \right] \left[1 + \frac{\pi(x)\zeta_D(x)\zeta_\xi(x)\delta(x)\lambda_b}{\beta\gamma(x)} \right]^{-1}, \end{aligned} \quad (40)$$

where the second line eliminates the recursive dependency of \hat{r} on ζ_B .

Regarding stock exposure to jumps, with probability $\pi(x)$ individual firm stockholders suffer a total loss, whereas with probability $1 - \pi(x)$ stock has positive residual value $P(\zeta_x(x)x, \tilde{\zeta}_i(x)Y_i) - B(x, Y_i)$, reflecting that firm value falls but bondholders take no losses outside of default. The conditional expected recovery rate for stocks reduces to

$$\zeta_S(x) = \frac{(1 - \pi(x))(\hat{r}(x)\delta(\zeta_x(x)x)\zeta_N(x) - \gamma(x)\beta)}{\hat{r}(x)\delta(x) - \gamma(x)\beta}, \quad (41)$$

for ζ_N defined in Appendix A.

Consistent with this exposure to jump risk, the equity premium is

$$[\zeta_\xi(x) + \zeta_S(x) - \zeta_S(x)\zeta_\xi(x) - 1]\lambda_b. \quad (42)$$

All of the preceding expressions for securities values and returns depend on the debt policy $\gamma(x)$. We consider two scenarios for determining equilibrium $\gamma(x)$.

As a baseline, we consider the policy $\gamma(x)$ chosen by value maximizing firm managers. This policy would arise in a complete contracting environment in which firm managers could contractually prohibit or deter idiosyncratic deviations from aggregate debt policy, and it has a closed form solution. Although the value maximizing equilibrium is also the solution to a central planner's problem, we take a decentralized solution approach for symmetry with the solution under limited commitment.

We then solve for $\gamma(x)$ with limited commitment, incorporating the distortion from controlling shareholder incentives introduced in Section 2.3.

3.1 Value Maximization

Suppose the manager of each firm i chooses a debt policy function $\gamma_i(x)$ to maximize firm value. Taking the SDF $\xi(x, Y)$ as given, firm i 's manager solves

$$\begin{aligned} \max_{\gamma_i} \mathbb{E}_t \left[\int_t^\infty \frac{\xi_s}{\xi_t} \delta(\gamma_i(x_s)) Y_{i,s} ds \right] \\ \equiv \max_{\gamma_i} p_i(x_t) Y_{i,t}, \end{aligned} \tag{43}$$

where $p_i(x)$ is the firm-specific price-output multiplier conditional on the state x and (implicitly) the manager's choice of γ_i .⁹

Although the manager's choice does not affect current output $Y_{i,t}$ it does affect the evolution of output. The manager's problem is equivalent to solving a Hamilton Jacobi Bellman (HJB) equation, summarized in Proposition 1. To keep expressions manageable, parameters to functions are suppressed, i.e., for the SDF we write ξ for $\xi(x, Y)$.

Proposition 1 *Under the assumption that managers maximize firm value, the manager's*

⁹Note that multiplier p_i is dependent on the function γ_i , not only on γ_i evaluated at x_t .

problem is equivalent to solving the HJB equation

$$\sup_{\gamma_i} \left\{ \xi \delta(\gamma_i) Y_i - \beta \xi p_i Y_i + (\xi^x p_i Y_i + \xi p_i' Y_i) x(1-x)(\lambda_b - \lambda_a) \right. \\ \left. + \xi^Y p_i Y_i \mu Y + \xi p_i \mu Y_i - \lambda_b (\xi p_i Y_i - \xi^J p_i^J Y_i^J) \right\} = 0. \quad (44)$$

The solution is characterized by the FOC

$$\frac{\beta \tau}{\alpha + 1} + \lambda_x (1 - \tau)(Z - 1) \gamma^\alpha + \tau(Z - 1) \left(\lambda_x + \frac{\beta}{\alpha + 1} \right) \gamma^{\alpha+1} = 0. \quad (45)$$

For integer values of α , the FOC is a polynomial, and admits a quasi-closed-form solution.

All proofs are in Appendix A.

3.2 Limited Commitment

We now solve for equilibrium debt policy when managers act in the best interest of current controlling shareholders, as described in Section 2.3. Such managers will trade off changes in total firm value against changes to the value of old debt. Firms commit to pay coupons at the prevailing corporate floating rate outside of bankruptcy and all debt has equal seniority, but deviations from aggregate debt policy affect the value of old debt by changing the probability of default.

We define the value of debt in firm i in terms of a unit debt multiplier,

$$b_i(x_t) = \mathbb{E}_t \left[\int_t^T \frac{\xi_s}{\xi_t} \hat{r}_s ds + \frac{\xi_T}{\xi_t} \frac{\hat{r}_{T-} P_{i,T}}{\gamma_{i,T-} Y_{i,T-} b_i(x_{T-})} \right], \quad (46)$$

which is a function of the state x and, implicitly, the debt policy γ_i of firm i . One unit of debt pays corporate floating rate \hat{r}_t until default at stochastic time T . Both the default time T and the recovery rate $\frac{\hat{r}_{T-} P_{i,T}}{\gamma_{i,T-} Y_{i,T-} b_i(x_{T-})}$ depend on the firm's debt policy γ_i .¹⁰ The expected

¹⁰We write subscript $T-$ for variable values the instant before default.

bond recovery rate conditional on default in state x is

$$\frac{\hat{r}(x)p_i(\zeta_x(x))\zeta_D(\gamma_i)}{b_i(x)\gamma_i}. \quad (47)$$

If $\gamma_i(x) = \gamma(x)$, $\forall x$, i.e., firm i follows the aggregate debt policy, then $b_i(x) = 1$, $\forall x$, by definition of the floating rate $\hat{r}(x)$, and $p_i(x) = p(x)$, $\forall x$ as earlier derived.

Two factors determine the extent to which firm managers diverge from value maximization: the amount of debt outstanding at the start of the current period (i.e., old debt), and the preference of current controlling owners for equity value maximization or bond value maximization. In choosing debt policy, managers treat both of these factors as exogenous fixed parameters. The scale of old debt is given by parameter Γ_i , and controlling owner preferences are given by parameter ρ_i .

Taking the SDF, ρ_i and Γ_i as given, firm i 's manager solves

$$\begin{aligned} & \max_{\gamma_i} \mathbb{E}_t \left[\int_t^\infty \frac{\xi_s}{\xi_t} \delta(\gamma_i(x_s)) Y_{i,s} ds \right] + \rho_i \Gamma_i \mathbb{E}_t \left[\int_t^T \frac{\xi_s}{\xi_t} \hat{r}_s ds + \frac{\xi_T}{\xi_t} \frac{\hat{r}_T - P_{i,T}}{\gamma_{i,T} - Y_{i,T} - b_i(x_{T-})} \right] \\ & \equiv \max_{\gamma_i} p_i(x_t) Y_{i,t} + \rho_i \Gamma_i b_i(x_t). \end{aligned} \quad (48)$$

The problem reduces to value maximization when $\rho_i = 0$. We define the manager's value function $V_i(x, Y_i, \rho_i \Gamma_i)$ as Equation (48) evaluated at its solution.

Once again the firm manager's problem is equivalent to solving an HJB equation, summarized in the following proposition.

Proposition 2 *Under the assumption of limited commitment, the manager's problem is equivalent to solving the HJB equation*

$$\begin{aligned} & \sup_{\gamma_i} \left\{ \xi(\delta(\gamma_i)Y_i + \rho_i \Gamma_i \hat{r}) - \beta \xi V_i + (\xi^x V_i + \xi V_i^x) x(1-x)(\lambda_b - \lambda_a) \right. \\ & \quad \left. + \xi^Y V_i \mu Y + \xi V_i^{Y_i} \mu Y_i - \lambda_b (\xi V_i - \xi^J V_i^J) \right\} = 0 \end{aligned} \quad (49)$$

The solution is characterized by the FOC

$$\underbrace{\tau}_{G_0} + \underbrace{\left(\frac{\lambda_x(\alpha+1)(1-\tau)G_V}{\delta(\gamma^J)} \right)}_{G_\alpha} \gamma^\alpha + \underbrace{\left(\frac{\lambda_x(\alpha+1)\tau G_V}{\delta(\gamma^J)} - \tau(1-Z) \right)}_{G_{\alpha+1}} \gamma^{\alpha+1} = 0.$$

For integer values of α , the FOC is polynomial taking coefficients as given. However, coefficients G_α and $G_{\alpha+1}$ depend on equilibrium capital structure following a jump. Therefore we solve the model numerically.

4 Numerical Example

Since our heterogeneous agent model does not have a closed-form solution, we study a loosely calibrated numerical example to illustrate the model's main mechanisms and informally assess their quantitative importance.

4.1 Calibration

Baseline model parameters are listed in Table 1. Table 2 lists relevant summary statistics from model and data. We take target data moments for the output growth rate, optimist growth forecast, pessimist growth forecast, riskless rate, equity premium, and stock return volatility from Baker et al. (2020).¹¹ The default rate is from Johnson et al. (2020).

The credit spread most relevant to our model is the average spread between corporate floating rate notes and the riskless rate. In the absence of liquidity premia, this is identical to the average corporate credit default swap rate. Data on credit spreads is less readily available than broad economic, equity, and Treasury rate data. We use the average monthly return on the iShares Floating Rate Bond ETF (FLOT) in excess of the 30-day Treasury bill yield, which is 79 basis points (bps). Data is from July 2011 to December 2020, from CRSP. This estimate is in line with the median 83 bps CDS premium reported by Berndt et al. (2018),

¹¹The optimist forecast is the average 75th percentile forecast from the Philadelphia Fed's Survey of Professional Forecasters, whereas pessimist forecast is the average 25th percentile forecast.

based on a sample from 2002-2015. Berndt et al. (2018) note substantial time-variation in CDS premia, with a median of 41 bps in 2006 versus 156 bps in 2009, so our estimate may not be representative of the long run mean.

To calibrate leverage, we roughly match the average ratio of corporate debt valuation to corporate equity valuation of nonfinancial corporations from Q4 1951 through Q4 2020, from the St. Louis Fed. The average ratio is 52.8%, but ranges from 102.4% in Q2 1982 to 26.8% in Q4 2020. Influencing the debt/equity ratio, our corporate tax rate of $\tau = 40\%$ roughly matches the average US statutory tax rate of 41.6% from 1952-2020, using data from the Tax Policy Center.¹² Parameter $Z = 0.35$, which implies that 65% of firm value is destroyed in default, is not directly observable. Glover (2016) estimates a 45% unconditional cost of default, implying a higher value of Z than we adopt.

In our model we approximate equity and corporate debt moments rather than output volatility, as our model with log utility is not capable of simultaneously approximating risky asset dynamics and output volatility. However we also report output volatility in Table 2, and list empirical output volatility from Johnson et al. (2020) as a point of reference.

Model results most relevant for calibration are simulated statistics from the heterogeneous agent model, given in column three. We also list statistics from economies dominated by either the optimistic agent a or pessimistic agent b , evaluated under their respective measures, in columns four and five. These economies provide useful points of reference, since observable variables in our heterogeneous economy converge to the optimistic economy of agent a as $x \rightarrow 1$, or to the pessimistic economy of agent b as $x \rightarrow 0$.

In order to calculate expectations in our heterogeneous economy, we must take a stand on the objective jump arrival rate and on the distribution of the main state variable x_t . We restrict the objective jump arrival rate λ_c to equal to the average of λ_a and λ_b , such that the drift of x_t is zero under the objective measure.¹³ From an initial Pareto weight of $x_0 = 0.5$,

¹²Effective tax rates are understood to be lower, roughly 27% based on the ratio of corporate tax receipts to pre-tax corporate profits from the St. Louis Fed. In this sense our parameter choice is on the high end.

¹³Formally we adopt objective measure c , with change of measure for agent a $\frac{d\eta_{a,t}}{\eta_{a,t}} = (\lambda_c - \lambda_a)dt +$

we simulate 100,000 paths of 50 years each at a monthly frequency, and report averages of annualized instantaneous moments in the terminal period under measure c . Moments from the homogeneous economies are also annualized instantaneous moments under each agent's respective measure.

Overall our model is quite parsimonious, with ten parameters total, but only seven free parameters used in calibration. We do not calibrate the control parameters θ and ψ as such, since these are the main parameters we explore in comparative statics, as their effects have not yet been studied in macroeconomic models. Instead our starting assumption is that stock carries control rights but bonds do not ($\psi = 1$), and that controlling influence is exactly proportional to the share of control rights ($\theta = 1$).

The statistics in Table 2 show that our calibration is close to the data in most instances, but clearly there are some tensions. The model has difficulty matching volatilities generally, and cannot simultaneously match stock and bond return volatility. Default rates and the credit spread are also somewhat high relative to data. Adopting a more flexible distribution for idiosyncratic output loss given jump would likely improve these model moments at the cost of increased solution complexity.¹⁴

The dispersion in beliefs in our calibration is also small, with the pessimist's λ_b only 10% higher than the optimist's λ_a . This implies a dispersion in expected GDP growth of about 0.8% in the heterogeneous agent model, versus about 1.4% in the data. Some studies of beliefs regarding financial disaster risk, e.g., Goetzmann et al. (2016), even support a larger dispersion in beliefs than that observed within the GDP growth estimates from the Survey of Professional Forecasters, our empirical point of reference. As we discuss in the next section, the amount of disagreement in our calibration nevertheless generates substantial variation in

$\left(\frac{\lambda_a}{\lambda_c} - 1\right) dJ_t^c$, change of measure for agent b $\frac{d\eta_{b,t}}{\eta_{b,t}} = (\lambda_c - \lambda_b)dt + \left(\frac{\lambda_b}{\lambda_c} - 1\right) dJ_t^c$, and resulting dynamic Pareto weight $x_t = \frac{\eta_{a,t}}{\eta_{a,t} + \eta_{b,t}}$. Solving the model under measure c does not alter equilibrium quantities conditional on the state x , which retains its interpretation as agent a 's Pareto weight and consumption share.

¹⁴Our power distribution has only one parameter, α , which we also restrict to integer values to allow use of a rootfinder in solving Equation (54) and Equation (60).

capital structure while preserving existence of equilibrium as we vary parameters in a series of comparative static exercises.

4.2 Characterization with baseline parameters

Our theoretical model allows novel analysis along two main dimensions. First, we can study the general equilibrium effects of disagreement among investors on corporate capital structure. Second, we can study the general equilibrium effects of limited commitment on capital structure when control rights are a function of endogenous portfolio holdings. To allow simultaneous consideration of both these aspects, we plot state-contingent statistics from our model with limited commitment and parameters in Table 1 alongside an otherwise identical economy with commitment. The economy with commitment is equivalent to setting $\theta = 0$, leaving other parameters unchanged from Table 1. Where relevant, all plots take expectations under the objective measure (λ_c).

Figure 2 shows how capital structure and related statistics vary with our main state variable x , the optimist's Pareto weight. We begin with the comparatively simple case of commitment to value-maximization, shown in dashed red, which corresponds to constant distortion coefficient $\rho = 0$. As x increases, the interest-output ratio γ increases approximately linearly from just above 18% to about 19%. The annualized default rate increases correspondingly from about 0.875% to about 0.95%. However the debt-equity ratio decreases by half, from about 80% when $x = 0$ and the economy is dominated by pessimists, to about 40% when $x = 1$ and the economy is dominated by optimists. The explanation is that although the interest share of output increases only marginally, investor perceptions of future growth relative to default risk are much more positive when the optimist dominates, boosting equity. Increased perceived growth also manifests in higher interest rates, depressing debt valuations.

Therefore, under commitment to value maximization, variation in the investor beliefs has substantial impact on the debt-equity ratio via the pricing kernel, but variation in the

firm's capital structure policy and objective default rate is small and approximately linear in average investor beliefs.

Results under limited commitment are shown as blue solid lines in Figure 2. In contrast with previous results, the interest-output ratio and default rate are nonmonotonic in x , with a wider range of variation. As x increases from 0 towards 1, the interest-output ratio initially declines from around 18% to around 16%, then rises to a peak of around 22% before declining again to around 19%. This variation tracks the distortion coefficient ρ .

Distortion peaks around $x = 0.25$, when the pessimist owns a controlling equity stake but has a strong portfolio tilt towards debt. A positive ρ implies the controlling owner would increase existing debt value at the expense of total firm value. The state-contingent equilibrium is sufficiently decreased interest-output ratio (γ) and default rate, relative to value maximization, that further reduction in γ would not boost the value of existing debt enough to offset the marginal decrease in total firm value.

Distortion takes its minimum value around $x = 0.7$, when the optimist owns a controlling equity stake and has a strong portfolio tilt towards equity. The logic is roughly the mirror image of the previously discussed case, with negative ρ leading to increased γ in equilibrium, relative to the value maximizing case.

Finally, when $\rho = 0$, the value maximization and limited commitment results intersect. This occurs three times: when the pessimist owns all debt and equity ($x = 0$), when the optimist owns all debt and equity ($x = 1$), and when each agent has exactly a 50% controlling interest in the firm. Although the two investors have very different individual objectives in the latter case, the firm's manager weighs them equally, and the overall distortion is zero.

Despite large changes in the magnitude and direction of variation in γ across two examples, the debt-equity ratio is similar under value maximization or limited commitment. The firm's capital structure decision has a second order influence on the valuation ratio relative to mean investor beliefs, which are identical across the two examples for each value of x .

In summary, there are three main takeaways from our capital structure results in Figure 2.

First, even modest differences in beliefs can generate variation in debt-equity ratios covering over 50% of the empirically observed range, and this variation has little to do with limited commitment frictions.¹⁵ Second, limited commitment amplifies variation in the interest-output ratio and objective default risk, with about sextuple the peak-to-trough range under limited commitment relative to value maximization.

Third, limited commitment changes how leverage correlates with the business cycle. As is apparent in Figure 2, the interest-output ratio γ is a good proxy for aggregate default risk in our model, whereas the debt-equity ratio is a poor proxy, so we focus on γ as a measure of leverage. Under value maximization, γ is moderately positively correlated with the business cycle, because x rises during booms (no jumps) and falls during busts (jumps).¹⁶ Under limited commitment, γ is more strongly positively correlated with the business cycle when x is between approximately 0.25 and 0.7, but it is negatively correlated with the business cycle in the intervals 0 to 0.25 and 0.7 to 1. As shown in Figure 1, if initially $x = 0.5$, then x is very likely to remain in the interval 0.25 to 0.7 even after 50 years, so the relationship between leverage and the business cycle could appear positive and approximately linear for a long period of time. Alternatively, an initial x around 0.25 or 0.7 would present an ambiguous or u-shaped relationship between γ and the business cycle, whereas x near 0 or 1 would present a negative relationship between γ and the business cycle.

A summary of discount rates and risk premia is given in Figure 3. In the two left panels, we can see that the riskless rate rises from around 0.6% when $x = 0$ to around 1.8% when $x = 1$, whereas the equity premium declines from around 8% when $x = 0$ to around 4.75% when $x = 1$. Overall the riskless rate and the equity premium behave similarly with or without commitment, with limited commitment introducing some curvature in r relative to the approximately linear value maximization example. The change in rates explains the declining debt-equity ratio in Figure 2, and the relative similarity of rates with or without

¹⁵Recall that we also assume log utility. Increased risk-aversion would likely increase the effect of a given level of disagreement, with or without commitment.

¹⁶The law of motion for x is the same with or without commitment.

commitment also clarifies why the the debt-equity ratio is similar across the two examples: for the most part, rates change because average investor beliefs change, not because capital structure changes.

However the right panels of Figure 3, plotting the corporate floating rate and the credit spread versus x , illustrate the impact of limited commitment on default risk. Whereas the credit spread hardly changes under value maximization, it varies from under 1% to almost 1.6% under limited commitment, following the wave-form pattern of γ . Changes in the credit spread dampen changes in the overall corporate rate when one of the two investor types is dominant, but amplify changes in the corporate rate when x is between 0.25 and 0.7.

The distortion coefficient ρ reflects differences between optimistic and pessimistic investor portfolios, and the apportionment of control rights. However the apportionment of control rights may itself alter equilibrium portfolios, either amplifying or dampening the direct effects of changes to our parameters θ and ψ . In terms of the allocation of control rights, the difference between our value maximization scenario, with $\theta = 0$, and our baseline example, with $\theta = 1$, is quite stark. However we can see in Figure 4 that portfolios are essentially identical across the two scenarios. Although we do not report the results here, we have explored alternative calibrations where changes in θ have larger feedback effects to portfolios.

We can also see in Figure 4 that although the optimist underweights bonds and the pessimist underweights stock, the difference is not stark.¹⁷ This is primarily because the beliefs of the optimist and pessimist differ by only 10%.

4.3 Comparative static examples

To better understand the effects of control rights with limited commitment, we conduct a conditional comparative static analysis by varying the two parameters governing the allocation of control rights, θ and ψ , in Figure 5 and Figure 6, respectively. Other parameters are per Table 1. Both figures show differences from the value maximizing equilibrium cor-

¹⁷The agents would have portfolios equal to their Pareto weights given identical beliefs, i.e., agent a 's portfolio would be a 45 degree line whereas agent b 's portfolio would be a negative 45 degree line.

responding to full commitment. Differencing the results can be thought of as removing the effects of disagreement absent distortions due to limited commitment. Figures 5 and 6 illustrate equilibrium conditional on the optimist's Pareto weight, x . The density of x remains the same as in Figure 1 despite varying parameters θ and ψ , because the dynamics of x depend only on parameters for the jump arrival rate.

In Figure 5, increasing θ corresponds to decreasing minority control rights. Results for $\theta = 0$, corresponding to value maximization, represent the point of reference and are identically zero in the plots. For $\theta > 0$, distortions due to limited commitment decrease the interest-output ratio γ when the pessimist has majority control, and increase γ when the optimist has majority control, with the shift in control occurring around $x = 0.45$ for all values $\theta > 0$. Increasing θ has two main effects on the distortion of γ .

First, the trough-to-peak range of the distortion to γ increases with θ , reaching around 10% for our maximum parameter value $\theta = 10$. This propagates to increased ranges for the default rate, equity premium, credit spread, and debt-to-equity ratio, also shown in Figure 5. For example, the trough-to-peak change in annualized default rate due to limited commitment is about 1% for $\theta = 10$, which is large relative to the roughly 1% mean default rate in our baseline calibration.

Second, the shift from the pessimist regime to the optimist regime becomes more abrupt as θ increases. When $\theta = 1$, γ takes its relative minimum when x is around 0.25 and its relative maximum when x is around 0.7. By contrast, when $\theta = 10$, γ takes its relative minimum when x is around 0.4 and its relative maximum when x is around 0.5. Since the dynamics of x remain the same for all values of θ , this implies that regime shifts occur more abruptly as θ increases. Another implication is that procyclical debt policy is less frequent as θ increases. Recall that x rises when there are no jumps (booms) and falls when a jump occurs (busts). The interest-output ratio γ is procyclical for x between 0.25 and 0.7 when $\theta = 1$, but is procyclical only for x between 0.4 and 0.5 when $\theta = 10$.

Therefore our model implies that, across otherwise identical economies, debt policy is

more volatile and more often countercyclical as minority control rights are diminished.

In Figure 5, smaller values of ψ correspond to weaker stockholder control rights. We consider values from $\psi = 0.6$, which means that 60% of control rights are allocated to stockholders and 40% to creditors, to $\psi = 1$, which means that all control rights are allocated to stockholders, as in our baseline calibration. Overall the shape and range of results is similar across all values of ψ . However there are two changes as stockholder control rights decrease.

First, results shift to the right. This is because pessimists, who tilt their portfolios towards debt, retain majority control for larger values of x when debt carries control rights. Second, results shift down as stockholder rights decrease. Relative to the value maximizing equilibrium, decreased stockholder rights worsen distortions when pessimists have majority control, but mitigate distortions when optimists have majority control. For example during times of pessimist control, the interest-output ratio is up to 2.5% below the value maximizing policy when $\psi = 0.6$, versus at most 2% below the value maximizing policy when $\psi = 1$. But during times of optimist control, the interest-output ratio is up to 3% above the value maximizing policy when $\psi = 1$, versus up at most 2.5% above the value maximizing policy when $\psi = 0.6$.

According to our model, an increase in creditor control rights may mitigate distortions from value maximization when optimists are in control, but it will exacerbate such distortions when pessimists are in control.

5 Conclusion

If the allocation of control rights to securities is to impact the behavior of firms, at least two conditions must be satisfied: ownership of the firms' securities must be heterogeneous, and some friction, such as incomplete contracting, must limit the ability of the firm's heterogeneous owners to commit to value maximization as a shared objective. In reality both of these conditions are easily satisfied, by obvious variation in securities ownership and investor

characteristics on the one hand, and by the legal impracticality of complete contracting on the other. Therefore it is very likely that the allocation of control rights matters for firm behavior, asset pricing, and ultimately macroeconomic dynamics.

We are unaware of existing theoretical work that models the essential elements described above in dynamic stochastic general equilibrium (DSGE). Indeed, a prerequisite for our particular study of control rights and aggregate debt dynamics is a DSGE model in which firms choose capital structure in the presence of investor heterogeneity *per se*, even with the objective of the firm taken as given. Therefore both the limited commitment and value maximization variants of our model are novel.

Several interesting findings emerge from our study. First, a reasonable hypothesis is that investor sentiment, captured in our model by the wealth share of the optimist relative to the pessimist, influences the amount of risk taken by firms, in the form of debt issuance in our case. If firms maximize value, there are some limitations to this hypothesis. Under value maximization, we show that shifts in sentiment can have a large impact on the debt-to-equity ratio via discount rates, but the interest-output ratio changes relatively little, and in the opposite direction, modestly increasing with optimist wealth while the debt-equity ratio drops by as much as 50%. Because firms increase credit risk as investor beliefs about such risk become more optimistic, sentiment alone has hardly any effect on credit spreads.

Introducing limited commitment, by contrast, leads to credit spreads that vary with sentiment by over 50% of their historical mean. In addition the relationship with sentiment becomes nonmonotonic, with procyclical and countercyclical regions. These two implications are related: it is because limited commitment can cause credit risk to move in the opposite direction of sentiment that credit spreads are more affected by changes in sentiment.

Having illustrated the impact of limited commitment relative to an appropriate baseline, we consider the implications of how control rights are allocated to securities, and how much influence minority owners wield. We find that allocating more control rights to creditors mitigates distortions from value maximization when optimists are in control, but exacer-

bates such distortions when pessimists are in control. It is ambiguous whether strengthened creditor control rights yields an overall benefit. By contrast strengthening minority influence brings the firm objective closer to value maximization in our setting, and minority protections have implications for aggregate debt dynamics. We find that debt policy is more volatile and more often countercyclical as minority control rights are diminished. This and other potentially testable implications of our model open avenues of future empirical research.

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Appendix

A Coefficients and Supporting Derivations

This appendix contains miscellaneous details omitted for brevity from the main text.

The unconditional expected recovery rate can be written $\zeta = \pi\zeta_D + (1 - \pi)\zeta_N$, where the expected recovery rate given default ζ_D and the expected recovery rate given no-default ζ_N are

$$\zeta_D = \frac{\alpha Z \gamma_{i,t-}}{\alpha + 1}, \quad (50)$$

$$\zeta_N = \frac{\alpha(1 - \gamma_{i,t-}^{\alpha+1})}{(\alpha + 1)(1 - \gamma_{i,t-}^\alpha)}. \quad (51)$$

The value of any firm i with current output Y_i is given by $P_i = P(x, Y_i)$, since

$$\begin{aligned} P_{i,t} &= \mathbb{E}_t \left[\int_t^\infty \frac{\xi_s}{\xi_t} \delta(x_s) Y_{i,s} ds \right] \\ &= (1 - x_t) \delta(x_t) Y_t \int_t^\infty e^{-\beta(s-t)} \mathbb{E}_t \left[(1 - x_s)^{-1} \frac{Y_{i,s}}{Y_s} \right] ds \\ &= \frac{1}{1 + \eta_t} \delta(x_t) Y_t \int_t^\infty e^{-\beta(s-t)} \left(\mathbb{E}_t^b \left[\frac{Y_{i,s}}{Y_s} \right] + \eta_t \mathbb{E}_t^a \left[\frac{Y_{i,s}}{Y_s} \right] \right) ds \\ &= \frac{\delta(x_t) Y_{i,t}}{\beta}. \end{aligned} \quad (52)$$

The second to last line uses \mathbb{E}_t^j , $j \in \{a, b\}$, to indicate expectations under the reference beliefs of agent b or under those of agent a . The last line follows from $\mathbb{E}_t^j \left[\frac{Y_{i,s}}{Y_s} \right] = \frac{Y_{i,t}}{Y_t}$, $j \in \{a, b\}$, since individual and aggregate output have the same drift, experience the same number of jumps, and the expected output jump size for any firm i is equal to the conditionally deterministic aggregate jump size under both a and b 's measures. (Recall that a and b disagree about the arrival rate of jumps, but agree on the distribution of losses conditional on a jump.)

Proof of Proposition 1.

The HJB equation under value maximization is

$$\sup_{\gamma_i} \left\{ \xi \delta(\gamma_i) Y_i - \beta \xi p_i Y_i + (\xi^x p_i Y_i + \xi p_i' Y_i) x(1-x)(\lambda_b - \lambda_a) \right. \\ \left. + \xi^Y p_i Y_i \mu Y + \xi p_i \mu Y_i - \lambda_b (\xi p_i Y_i - \xi^J p_i^J Y_i^J) \right\} = 0 \quad (53)$$

where

$$\xi^x = \frac{\delta(\gamma(x)) - (1-x)\delta'(\gamma(x))\gamma'(x)}{(1-x)\delta(\gamma(x))} \xi, \\ \xi^Y = \frac{\xi}{Y}, \\ \xi^J = \xi(x^J, \zeta(\gamma(x))Y), \\ p_i^J = p_i(x^J), \\ Y_i^J = \zeta(\gamma_i)Y_i, \\ x^J = \zeta_x(x)x.$$

In the HJB equation, only the terms for the dividend stream (first term) and the jump outcome (last term) directly involve the control variable γ_i . The FOC is

$$\xi \tau Y_i + \lambda_b \xi^J p_i(x^J) \zeta'(\gamma_i) Y_i = 0, \quad (54)$$

where the first derivative of the expected recovery rate is

$$\zeta'(\gamma_i) = \alpha(Z-1)\gamma_i^\alpha. \quad (55)$$

Reducing the FOC to model primitives and simplifying, we have

$$\frac{\tau}{\delta(\gamma)} + \frac{\lambda_b(1-x)p_i(x^J)\zeta'(\gamma_i)}{(1-x^J)\delta(\gamma^J)\zeta(\gamma)} = 0. \quad (56)$$

We now impose the equilibrium fixed point requirement that all firms adopt the same optimal policy, i.e., $\gamma_i(x) = \gamma(x)$, which in turn implies that $p_i(x) = p(x)$. Substituting for these conditions in the FOC leads to

$$\frac{\tau}{\delta(\gamma)} + \frac{\lambda_x \zeta'(\gamma)}{\beta \zeta(\gamma)} = 0, \quad (57)$$

where $\lambda_x = \lambda_a x + \lambda_b(1-x)$ is the Pareto-weighted belief about jump risk.

Substituting λ_x , reducing to model primitives, and rearranging terms, the FOC is

$$\begin{aligned}
\frac{\tau}{\delta(\gamma)} + \frac{\lambda_x \zeta'(\gamma)}{\beta \zeta(\gamma)} &= 0, \\
\frac{\tau}{((1-\tau) + \tau\gamma)} + \frac{\lambda_x \alpha (Z-1) \gamma^\alpha}{\beta \frac{\alpha}{\alpha+1} (1 - (1-Z)\gamma^{\alpha+1})} &= 0, \\
\frac{\beta\tau}{\alpha+1} (1 - (1-Z)\gamma^{\alpha+1}) + \lambda_x (Z-1) ((1-\tau) + \tau\gamma) \gamma^\alpha &= 0, \quad (58) \\
\frac{\beta\tau}{\alpha+1} + \lambda_x (Z-1) (1-\tau) \gamma^\alpha + \frac{\beta\tau}{\alpha+1} ((Z-1)\gamma^{\alpha+1}) + \lambda_x (Z-1) \tau \gamma^{\alpha+1} &= 0, \\
\frac{\beta\tau}{\alpha+1} + \lambda_x (1-\tau) (Z-1) \gamma^\alpha + \tau (Z-1) \left(\lambda_x + \frac{\beta}{\alpha+1} \right) \gamma^{\alpha+1} &= 0.
\end{aligned}$$

For integer values of α , the FOC is a polynomial, and admits a quasi-closed-form solution.¹⁸ ■

Proof of Proposition 2.

With limited commitment, the HJB equation is

$$\begin{aligned}
\sup_{\gamma_i} \left\{ \xi(\delta(\gamma_i)Y_i + \rho_i \Gamma_i \hat{r}) - \beta \xi V_i + (\xi^x V_i + \xi V_i^x) x(1-x)(\lambda_b - \lambda_a) \right. \\
\left. + \xi^Y V_i \mu Y + \xi V_i^{Y_i} \mu Y_i - \lambda_b (\xi V_i - \xi^J V_i^J) \right\} = 0 \quad (59)
\end{aligned}$$

where

$$\begin{aligned}
\xi^x &= \frac{\delta(\gamma(x)) - (1-x)\delta'(\gamma(x))\gamma'(x)}{(1-x)\delta(\gamma(x))} \xi, \\
\xi^Y &= \frac{\xi}{Y}, \\
V_i^x &= p_i'(x)Y_i + \rho_i \Gamma_i b_i'(x), \\
V_i^{Y_i} &= p_i(x), \\
\xi^J &= \xi(x^J, \zeta(\gamma(x))Y), \\
V_i^J &= p_i(x^J)\zeta(\gamma_i)Y_i + \rho_i \Gamma_i \left((1 - \pi(\gamma_i))b_i(x^J) + \pi(\gamma_i) \frac{\hat{r}(x)p_i(x^J)}{b_i(x)} \frac{\zeta_D(\gamma_i)}{\gamma_i} \right), \\
x^J &= \zeta_x(x)x.
\end{aligned}$$

In the HJB equation, only the terms for the dividend stream (first term) and the jump outcome (last term) directly involve the control variable γ_i . The FOC is

$$\xi \tau Y_i + \lambda_b \xi^J \frac{\partial V_i^J}{\partial \gamma_i} = 0. \quad (60)$$

¹⁸In practise we use a software root-finder to solve the polynomial, and numerically verify that only one economically relevant solution exists.

The derivative of the jump term wrt γ_i is

$$\frac{\partial V_i^J}{\partial \gamma_i} = p(x^J) \zeta'(\gamma_i) Y_i + \rho_i \Gamma_i \pi'(\gamma_i) (\zeta_D^B - b(x^J)),$$

where

$$\begin{aligned} \pi'(\gamma_i) &= \alpha \gamma_i^{\alpha-1} \\ \zeta_D^B &= \frac{\hat{r}(x) p_i(x^J)}{b_i(x)} \frac{\zeta_D(\gamma_i)}{\gamma_i} = \left(\frac{\hat{r}(x) p_i(x^J)}{b_i(x)} \right) \left(\frac{\alpha Z}{\alpha + 1} \right). \end{aligned}$$

We focus on an equilibrium in which all firms follow the same debt policy. In any such equilibrium, $\gamma_i(x) = \gamma(x)$, $p_i(x) = p(x)$, $b_i(x) = 1$, and $\Gamma_i = \frac{\gamma(x) Y_i}{\hat{r}(x)}$. Further, since all firms are ex-ante identical, agents optimally hold fully diversified portfolios of bonds and stocks, and each agent's relative holdings of stock and bonds in any firm i mirror his relative holdings of diversified stock and bond portfolios. Therefore distortion coefficient $\rho_i = \rho(x)$, where $\rho(x)$ is defined per Equation (27), portfolios as a fraction of wealth ($\phi_B^j(x)$ and $\phi_S^j(x)$, $j \in \{a, b\}$) follow Equation (19), and each agent's fractional ownership of stock and bonds is

$$\hat{\phi}_B^j(x) = \frac{X_j(x, Y) \phi_B^j(x)}{B(x, Y)}, \quad (61)$$

$$\hat{\phi}_S^j(x) = \frac{X_j(x, Y) \phi_S^j(x)}{S(x, Y)}, \text{ where } j \in \{a, b\}. \quad (62)$$

Substituting the aforementioned equilibrium values and $\gamma_i = \gamma$ into the FOC, we have

$$\frac{\tau}{\delta(\gamma)} + \frac{\lambda_x}{\delta(\gamma^J) \zeta(\gamma)} \frac{1}{Y_i} \frac{\partial V_i^J}{\partial \gamma_i} = 0, \quad (63)$$

where

$$\begin{aligned} \frac{1}{Y_i} \frac{\partial V_i^J}{\partial \gamma_i} &= p(x^J) \zeta'(\gamma) + \rho(x) \frac{\gamma}{\hat{r}(x)} \pi'(\gamma) (\zeta_D^B - 1), \\ \pi'(\gamma) &= \alpha \gamma^{\alpha-1} \\ \zeta_D^B &= \hat{r}(x) p(x^J) \frac{\alpha Z}{\alpha + 1}. \end{aligned}$$

With some further manipulation, equilibrium γ in state x solves a polynomial equation similar to Equation (45), but with coefficients that depend on capital structure following a jump, i.e., $\gamma(x^J)$.

The derivative of the jump term can be written

$$\frac{1}{Y_i} \frac{\partial V_i^J}{\partial \gamma_i} = \alpha \gamma^\alpha \underbrace{\left(p(x^J)(Z - 1) + \frac{\rho(x)}{\hat{r}(x)} (\zeta_D^B - 1) \right)}_{G_V}.$$

The FOC can then be written

$$\begin{aligned}
& \tau \zeta(\gamma) + \frac{\lambda_x \delta(\gamma)}{\delta(\gamma^J)} \frac{1}{Y_i} \frac{\partial V_i^J}{\partial \gamma_i} = 0, \\
& \tau \frac{\alpha}{\alpha+1} (1 - (1-Z)\gamma^{\alpha+1}) + \frac{\lambda_x((1-\tau) + \tau\gamma)}{\delta(\gamma^J)} \frac{1}{Y_i} \frac{\partial V_i^J}{\partial \gamma_i} = 0, \\
& \tau \frac{\alpha}{\alpha+1} (1 - (1-Z)\gamma^{\alpha+1}) + \frac{\lambda_x((1-\tau) + \tau\gamma)}{\delta(\gamma^J)} \alpha \gamma^\alpha G_V = 0, \\
& \tau (1 - (1-Z)\gamma^{\alpha+1}) + \frac{\lambda_x(\alpha+1)G_V((1-\tau)\gamma^\alpha + \tau\gamma^{\alpha+1})}{\delta(\gamma^J)} = 0, \\
& \underbrace{\tau}_{G_0} + \underbrace{\left(\frac{\lambda_x(\alpha+1)(1-\tau)G_V}{\delta(\gamma^J)} \right)}_{G_\alpha} \gamma^\alpha + \underbrace{\left(\frac{\lambda_x(\alpha+1)\tau G_V}{\delta(\gamma^J)} - \tau(1-Z) \right)}_{G_{\alpha+1}} \gamma^{\alpha+1} = 0.
\end{aligned}$$

■

	Parameter	Value
Drift of dividends	μ	0.115
Jump arrival rate, agent a	λ_a	0.25
Jump arrival rate, agent b	λ_b	0.275
Jump arrival rate, econometrician c	λ_c	0.2625
Output recovery distribution parameter	α	2
Default recovery rate	Z	0.35
Corporate tax rate	τ	0.4
Subjective discount rate	β	0.03
Shareholder control rights	ψ	1
Majority control coefficient	θ	1

Table 1: Parameter values. The table reports the baseline parameter values used in our numerical examples.

	Data	Heterogeneous Econ.	Optimist Econ.	Pessimist Econ.
Output growth rate (%)	2.000	2.663	3.092	2.261
Optimist growth forecast (%)	3.030	3.084	3.092	NA
Pessimist growth forecast (%)	1.620	2.242	NA	2.261
Corporate tax rate (%)	41.630	40.000	40.000	40.000
Default rate (%)	0.870	1.011	0.906	0.911
Debt/equity ratio (%)	52.83	54.135	39.517	79.819
Riskless rate (%)	0.800	1.145	1.831	0.588
Credit spread (%)	0.79	1.324	1.151	1.241
Equity premium (%)	6.600	6.127	4.649	8.072
Output volatility (%)	3.060	17.248	16.816	17.617
Stock return volatility (%)	14.530	25.788	22.858	30.425
Bond return volatility (%)	1.890	1.713	1.528	1.571

Table 2: Summary statistics. The table reports summary statistics to roughly calibrate our numerical examples. Data moments are from a variety of sources described in the text. The Optimist Economy column gives results for an economy with only our optimistic agent a present, with moments evaluated under his beliefs λ_a . The Pessimist Economy column gives results for an economy with only our pessimistic agent b present, with moments evaluated under his beliefs λ_b . Model moments are annualized instantaneous moments. Both the pessimist and the optimist participate in our Heterogeneous Economy, for which moments are computed by simulating 100,000 paths of 50 years each at a monthly frequency under objective measure $\lambda_c = \frac{\lambda_a + \lambda_b}{2}$. We assume an initial state of $x_0 = 0.5$, assigning each agent equal initial wealth and consumption share, and report averages of the instantaneous annualized moments in the terminal simulated state.

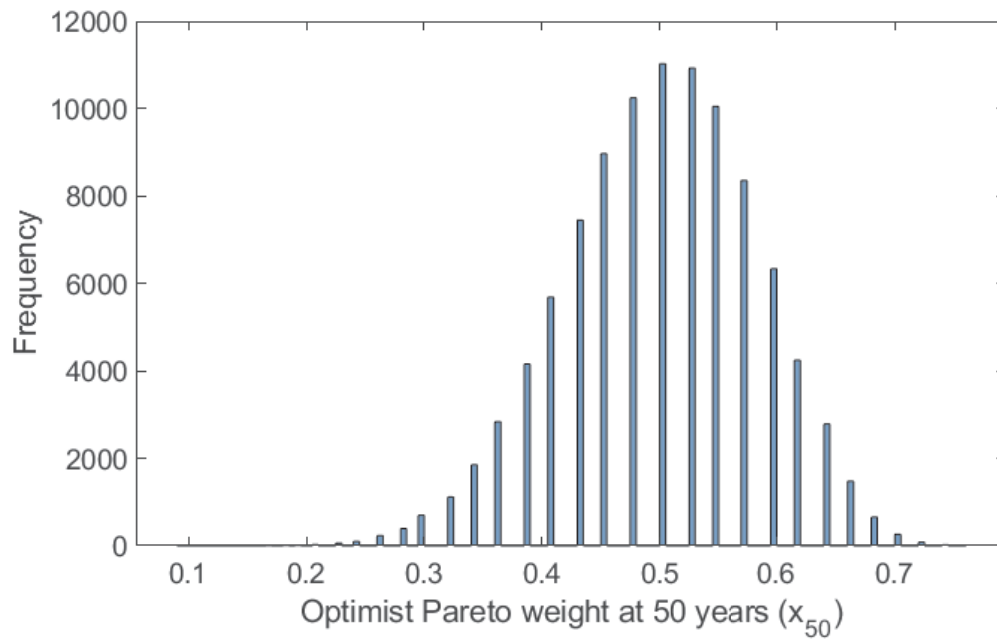


Figure 1: Simulated distribution of optimist's Pareto weight. The histogram shows the simulated distribution of the optimist's Pareto weight x_t after 50 years. We simulate 100,000 paths at a monthly frequency from an initial state of $x_0 = 0.5$. All parameters are per Table 1.

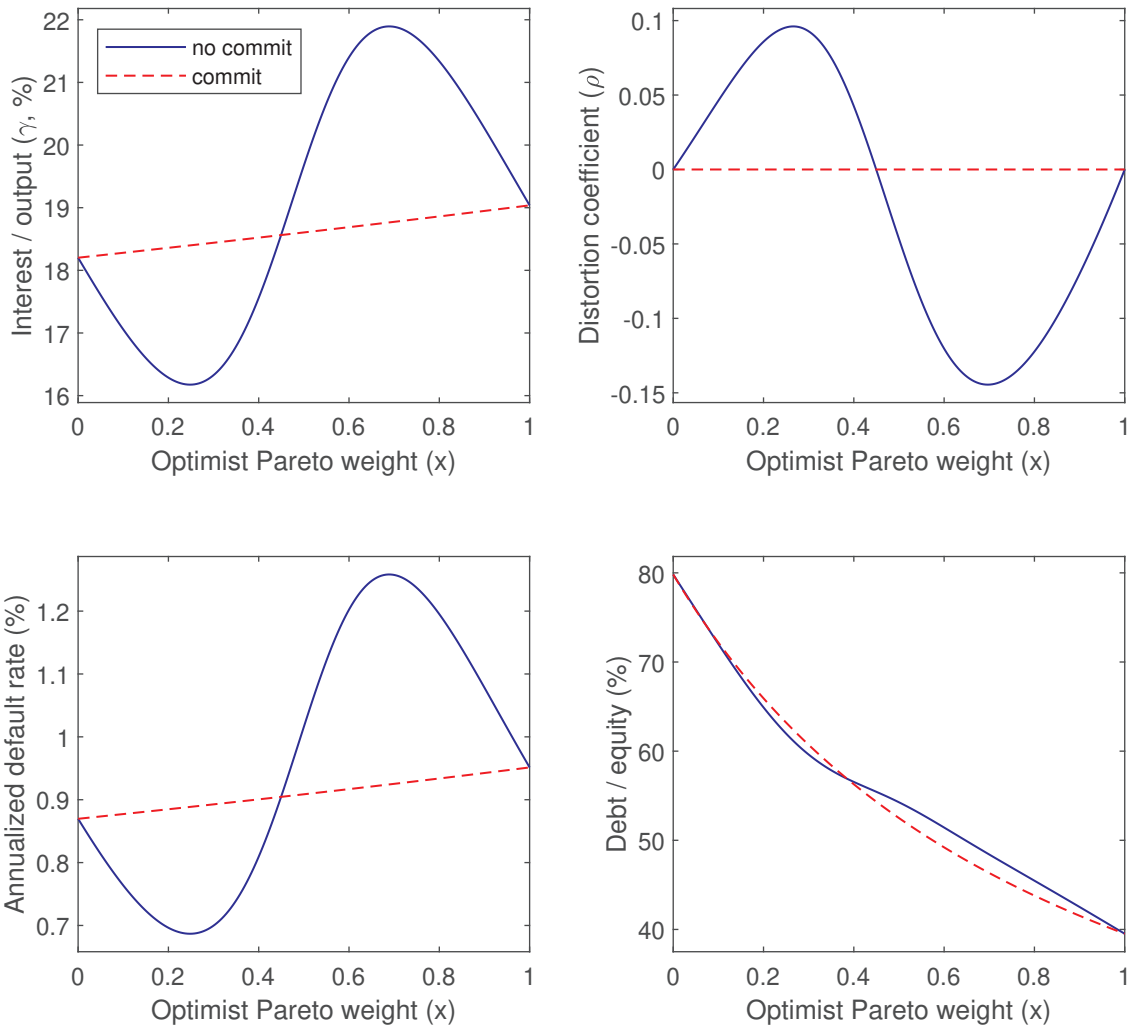


Figure 2: Capital structure, default rate, and distortion coefficient. The blue solid lines show results under limited commitment, whereas the red dashed lines show results when managers can commit to value maximization. All parameters are per Table 1.

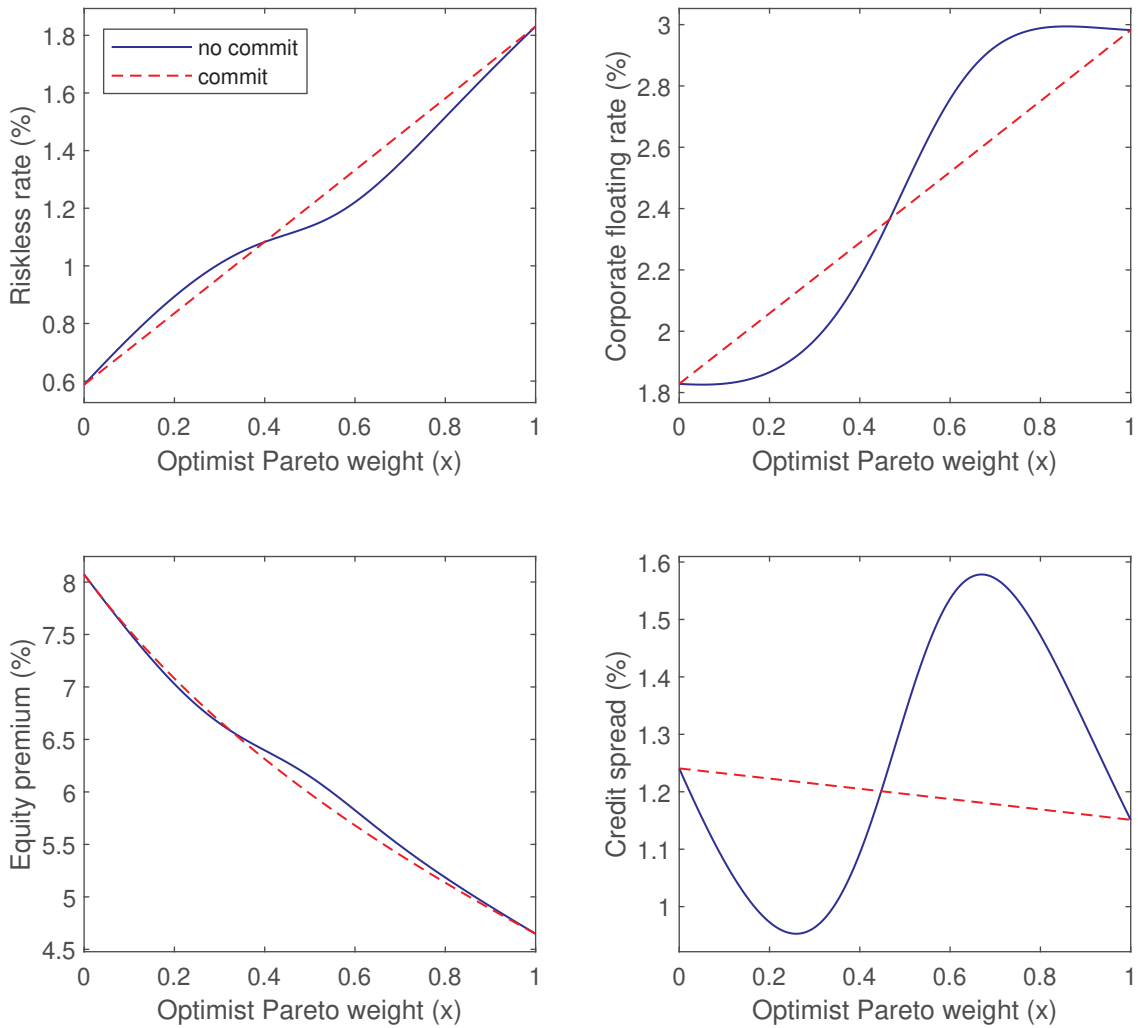


Figure 3: Expected returns and interest rates. The blue solid lines show results under limited commitment, whereas the red dashed lines show results when managers can commit to value maximization. All parameters are per Table 1.

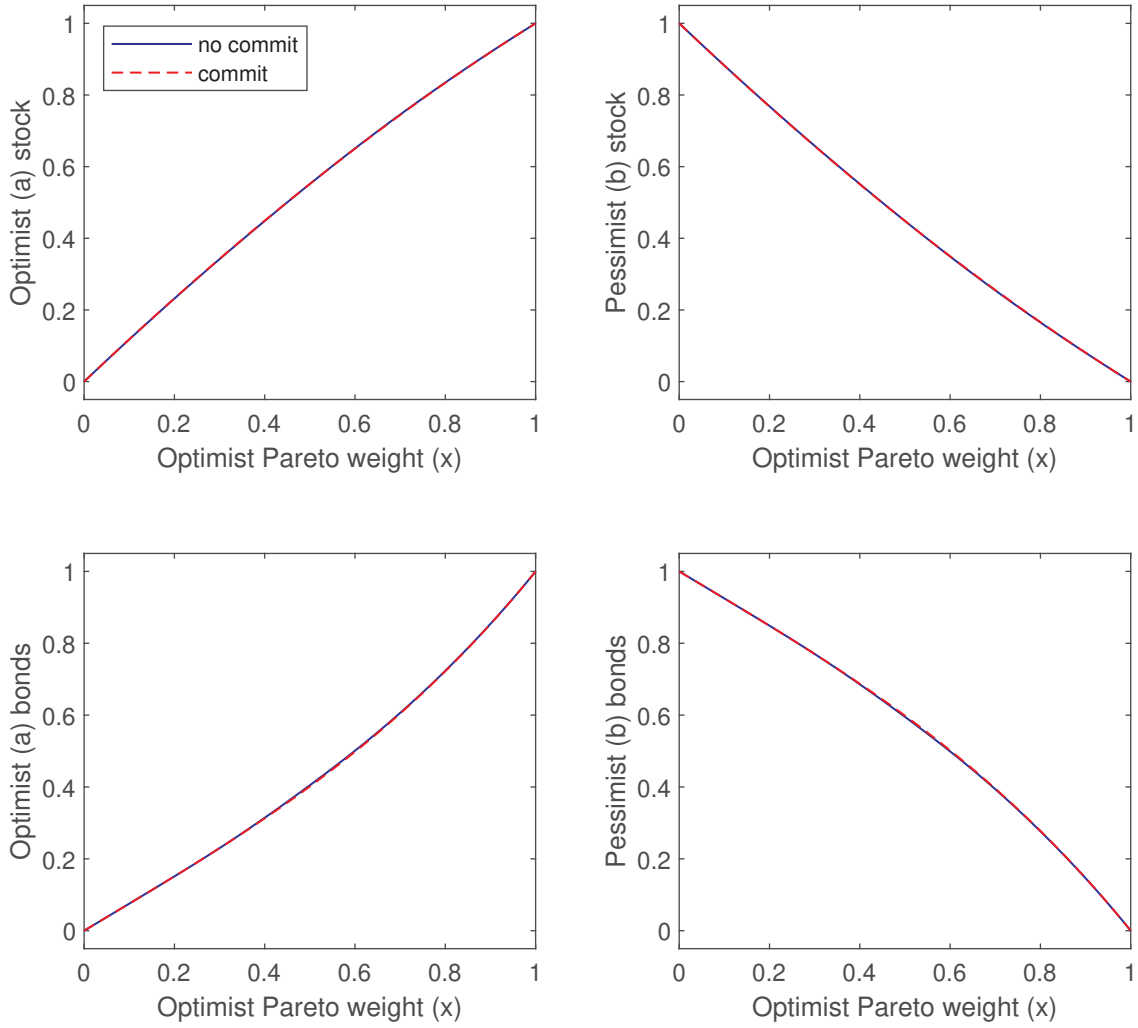


Figure 4: Portfolios The blue solid lines show results under limited commitment, whereas the red dashed lines show results when managers can commit to value maximization. All parameters are per Table 1.

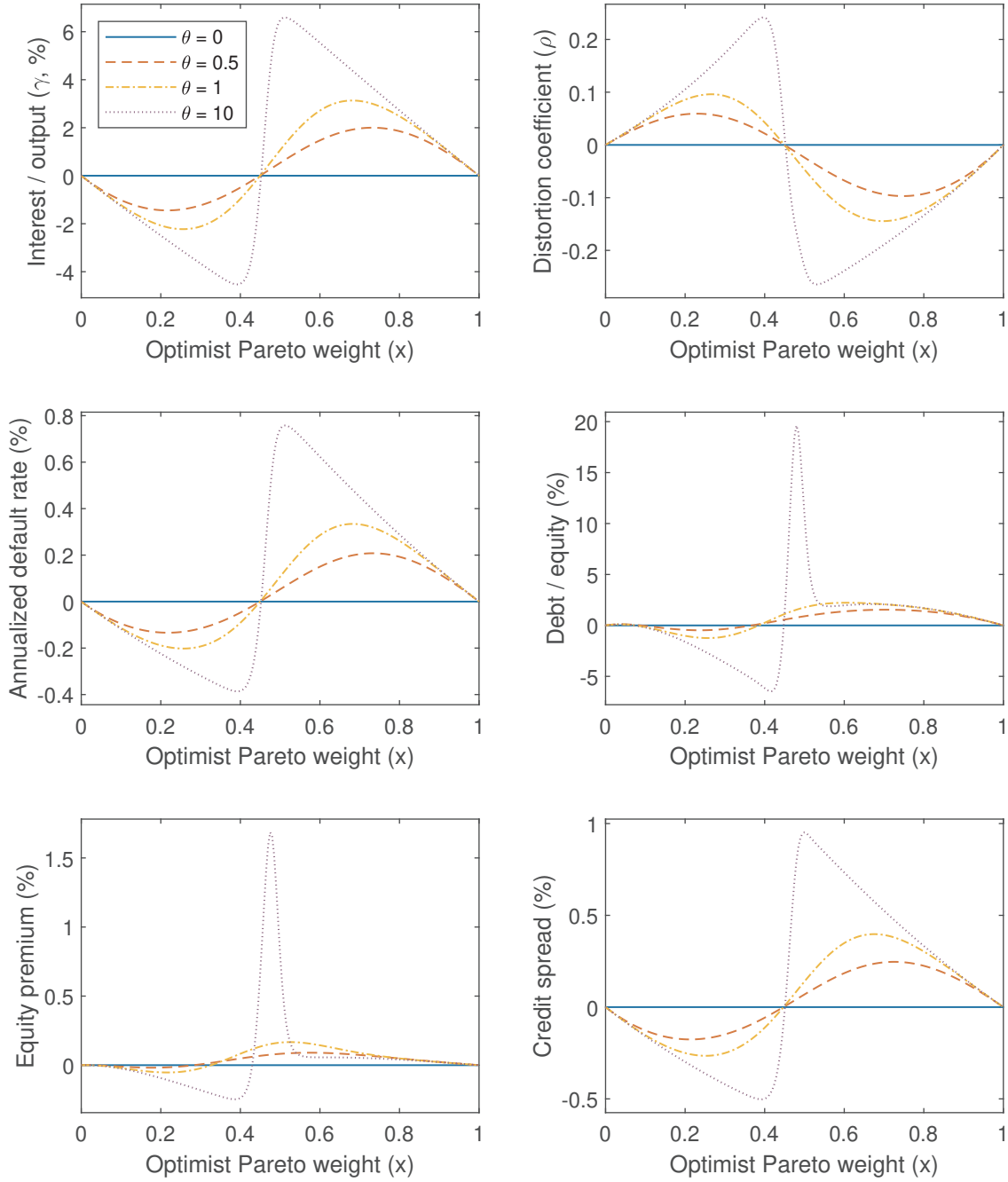


Figure 5: Vary minority control rights (θ). The figure shows the effect of minority control rights parameter θ on the limited commitment equilibrium relative to the full commitment (value maximization) equilibrium. Debt policy γ , distortion coefficient ρ , riskless rate r , and the corporate yield spread are plotted versus the Pareto share of agent a (x) for a range of values of θ , as given in the legend in the top left panel. All parameters except θ are per Table 1.

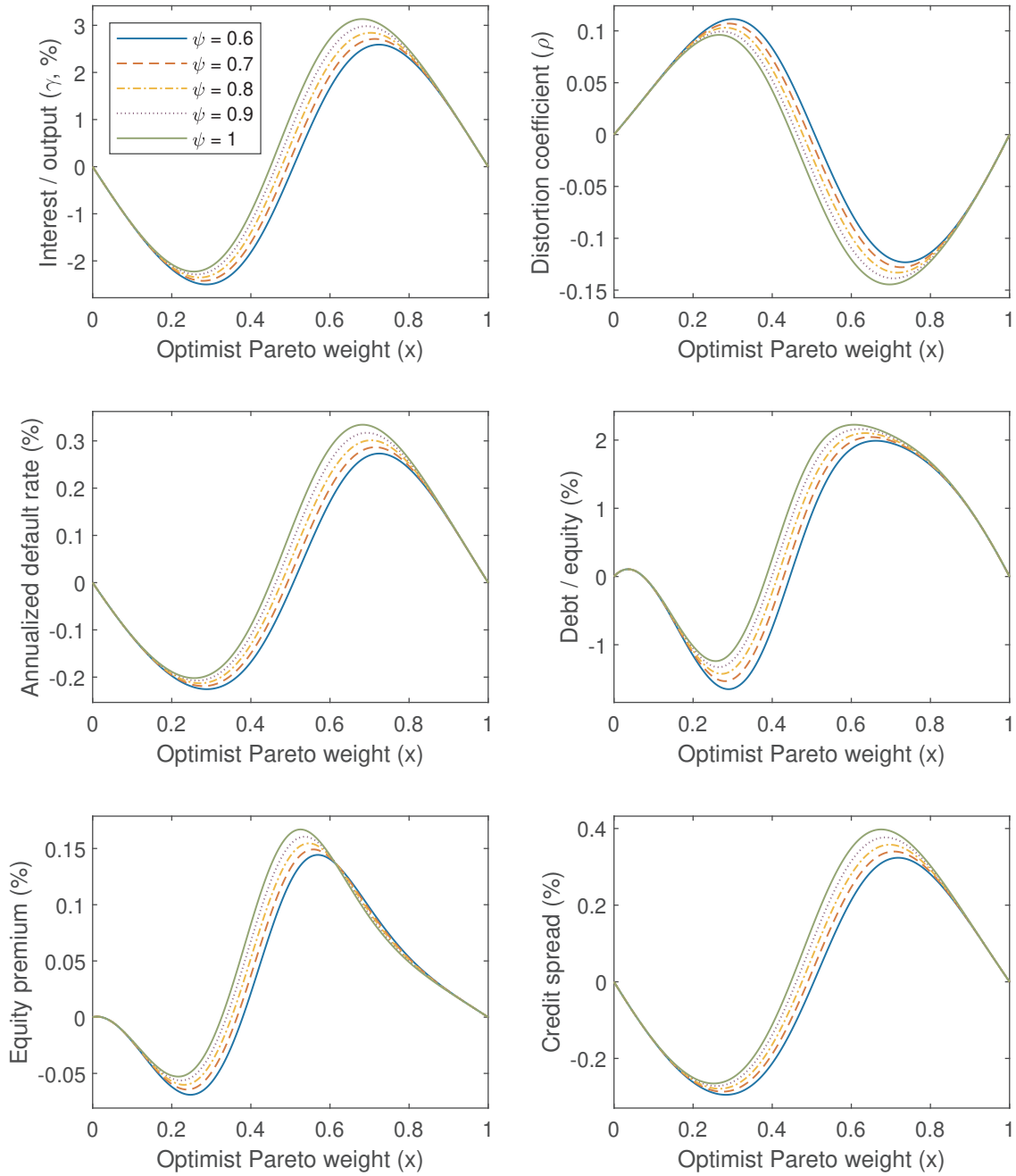


Figure 6: Vary shareholder control rights (ψ). The figure shows the effect of shareholder control rights parameter ψ on the limited commitment equilibrium relative to the full commitment (value maximization) equilibrium. Debt policy γ , distortion coefficient ρ , riskless rate r , and the corporate yield spread are plotted versus the Pareto share of agent a (x) for a range of values of ψ , as given in the legend in the top left panel. All parameters except ψ are per Table 1.